Euclidean Twistor Unification and the Twistor $\mathbf{P}^1$

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Note: These slides at
`https://www.math.columbia.edu/~woit/utdallas.pdf`
For more details, see
`https://arxiv.org/abs/2104.05099`

Background: fundamental physics

Our best theory of fundamental physics has three somewhat different components:

1. A four-dimensional space-time M with a pseudo-Riemannian metric of signature $(3,1)$. The dynamics of the metric are given by Einstein's equations (theory of general relativity).
2. A principal $SU(3) \times SU(2) \times U(1)$ bundle with connection over M. The dynamics of the connection are given by the Yang-Mills equations.
3. Matter fields on M with values that transform as space-time spinors and a specific set of representations of $SU(3) \times SU(2) \times U(1)$. Dynamics is given by the Dirac equation.

For 2 and 3 we have a consistent quantum theory (the Standard Model), for 1 only a consistent classical theory. We would like a "unified theory", a quantum theory bringing together $1, 2, 3$.

A proposal: twistor geometry and Euclidean signature

Penrose (1967) gave a very different formulation of four-dimensional space time, in which a space-time point is a complex 2-plane in a "twistor space" $\mathbb{C}^4$. Physicists have studied this intensively for Minkowski signature space-time.

Mathematicians (beginning with Atiyah, 1977) have also studied this, but for Euclidean signature space-time.

A question

Can one make progress on unification by using twistor geometry to describe space-time, taking as fundamental Euclidean signature space-time and analytically continuing in complex time to Minkowski signature?

The free particle

Consider a free particle moving in one spatial dimension. The Hamiltonian operator (units such that $\hbar = 1$) acts on wave-functions

$$H = -\frac{1}{2m}\frac{d^2}{dx^2}$$

Solutions to the Schrödinger equation $i\frac{d}{dt}\psi = H\psi$ are given by

$$e^{-iHt}\psi(x,0)$$

Taking Fourier transforms in x, on $\widetilde{\psi}(p,t)$ the operator H is the diagonal operator $\frac{p^2}{2m}$, and e^{-iHt} is multiplication by $e^{-i\frac{p^2}{2m}t}$. The inverse Fourier transform then gives

$$\psi(x',t) = \int_{-\infty}^{\infty} K(x',x,t)\psi(x,0)$$

where

$$K(x',x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\frac{p^2}{2m}t} e^{ip(x'-x)}dp$$

Imaginary time and the propagator

One way to make sense of K (the propagator) as a distribution is to introduce a complex time variable $z = t + i\tau$ and then define K as the boundary value at $\tau = 0$ of something holomorphic for $\tau < 0$

$$K(x', x, t) = \lim_{\tau \to 0^-} K(x', x, z) = \lim_{\tau \to 0^-} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\frac{p^2}{2m}z} e^{ip(x'-x)} dp$$

One can define K by analytic continuation from its values for $z = i\tau, \tau < 0$, where it is the well-known heat kernel, solving the heat equation rather than the Schrödinger equation.

Path integrals

In the path integral formalism one tries to define an integral over paths

$$K(x', x, t) = \int_{q(0)=x, q(t)=x'} e^{-i\int_0^t \frac{1}{2}m\dot{q}^2 dt'} \mathcal{D}q(t')$$

There is no possible well-defined measure on the space of paths that will make sense of this, but replacing t by $i\tau$ one can define such integrals. In general (passing to quantum field theories and integrals over fields)

Defining QFTs by path integrals

Looking at the path integrals

$$\int F[\phi] e^{iS_M(\phi)} d\phi \text{ versus } \int F[\phi] e^{-S_E(\phi)} d\phi$$

If you do rigorous mathematics you can't make sense of the first, can sometimes make sense of the second (ask a mathematical physicist). If you do numerical calculations, same thing (ask a lattice gauge theorist).

Relativistic quantum field theory and Euclidean space-time

In relativistic quantum field theory one treats time and space on an equal footing, using the Minkowski metric on $x = (t, \boldsymbol{x})$

$$|x|_M^2 = |\boldsymbol{x}|^2 - t^2$$

quantizing fields $\phi(x)$ to treat arbitrary numbers of particles. Every quantum field theory textbook explains that there's a problem even in free field QFT. Computing the propagator involves taking the Fourier transform of

$$\frac{i}{\omega_{\boldsymbol{p}}^2 - E^2}$$

where $\omega_{\boldsymbol{p}}^2 = |\boldsymbol{p}|^2 + m^2$. To do this you have to decide what to do about the poles $E = \pm\omega_{\mathbf{p}}$. The physically sensible answer corresponds to analytically continuing from imaginary time τ, defining the theory in Euclidean space-time with

$$|x|_E^2 = |\boldsymbol{x}|^2 + \tau^2$$

Four-dimensional geometry and 2x2 complex matrices

Complexifying not just time, but space-time, one can do four-dimensional complex geometry by identifying $\mathbb{C}^4$ with 2x2 complex matrices

$$(z_0, z_1, z_2, z_3) \leftrightarrow z = z_0\mathbf{1} - i(z_1\sigma_1 + z_2\sigma_2 + z_3\sigma_3) = \begin{pmatrix} z_0 - iz_3 & -z_2 - iz_1 \\ z_2 - iz_1 & z_0 + iz_3 \end{pmatrix}$$

(here σ_j are the Pauli matrices) and defining

$$|z|^2 = \det z$$

Pairs $g_L, g_R \in SL(2,\mathbb{C}) \times SL(2,\mathbb{C}) = Spin(4,\mathbb{C})$ act preserving $|z|$ by

$$z \to g_L z g_R^{-1}$$

We are interested in real forms of this (real 4d vector spaces that give the above after complexification).

Real forms

Three real forms and the corresponding groups (real forms of $Spin(4,\mathbb{C})$) are

- $(2,2)$ signature inner product: $Spin(2,2) = SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$, using $g_L, g_R \in SL(2,\mathbb{R})$.
- $(3,1)$ signature inner product: $Spin(3,1) = SL(2,\mathbb{C})$, using $g_R = (g_L^\dagger)^{-1}$
  This is Minkowski space-time.
- $(4,0)$ signature inner product: $Spin(4,0) = SU(2) \times SU(2)$, using $g_L, g_R \in SU(2)$.
  This is Euclidean space-time.

Our interest will be in the Minkowski and Euclidean cases, together with the analytic continuation relating them.

Euclidean signature and quaternions

In Euclidean signature, can use quaternions instead of complex matrices

$$(x_0, x_1, x_2, x_3) \leftrightarrow x = x_0\mathbf{1} + x_1\boldsymbol{i} + x_2\boldsymbol{j} + x_3\boldsymbol{k}$$

with $|x|^2 = x\overline{x}$ and rotations given by pairs q_L, q_R of unit length quaternions.

$$x \rightarrow q_L x q_R^{-1}$$

Note that when we do this, we now have a conjugation operation (changing sign of $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$).

Spinor geometry

Thinking of four-dimensional vectors as 2 x 2 complex matrices, they are linear maps from one $\mathbb{C}^2$ (called the (half)-spinor space S_R) to another $\mathbb{C}^2$ (called the (half)-spinor space S_L). Corresponding to the action on vectors

$$x \rightarrow g_L x g_R^{-1}$$

we have actions on S_R, S_L by

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_R \in S_R \rightarrow g_R \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_R \in S_R$$

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_L \in S_L \rightarrow g_L \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_L \in S_L$$

Analytically continuing spinors is problematic

- In Euclidean space, g_R and g_L are independent $SU(2)$ matrices.
- In Minkowski space, $g_R \in SL(2, \mathbb{C})$ and g_L is determined by g_R $(= (g_R^{-1})^\dagger)$.

Twistor theory

Twistor geometry is a different way of thinking about the geometry of space-time, first proposed in 1967 by Penrose. It naturally provides a joint complexification of Minkowski and Euclidean space-time and a way to look at analytic continuation between them. In twistor theory one takes as fundamental twistor space $T = \mathbb{C}^4$ (or its projective version $PT = \mathbb{C}P^3$, the complex lines in T).

Historical note: Penrose writes that he had the basic idea in late 1963. At that time he was at UT Austin, and after the Kennedy assasination he planned to meet up with colleagues from UT Dallas (Ivor Robinson, Wolfgang Rindler, Istvan Ozsvath) for a road trip to San Antonio and elsewhere. After spending time with them he was being driven back to Austin by Ozsvath, who evidently was not very talkative, and during this ride he had the idea.

Why twistor space?

Two arguments for twistors

In twistor theory spinors are tautological

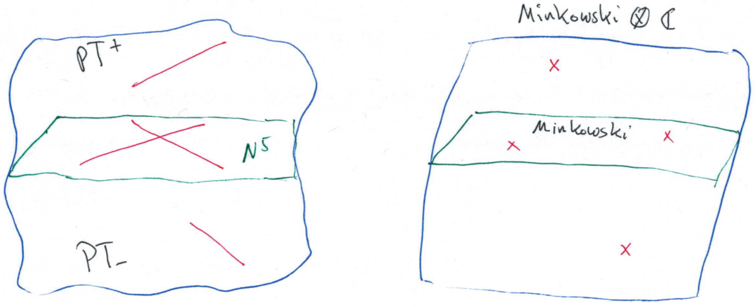
Points of space-time will correspond to a $\mathbb{C}^2 \subset T$ (or projectively a $\mathbb{C}P^1 \subset \mathbb{C}P^3$) tautologically giving the fiber S_R of the half-spinor bundle.

The $\mathbb{C}P^1$ is directly experienced

In Minkowski space-time, the $\mathbb{C}P^1$ describing a space-time point corresponds to the sphere of directions of light rays one sees when one opens an eye.

Minkowski space-time twistors: a picture

Minkowski space-time is defined by a nondegenerate signature $(2, 2)$ Hermitian form Φ on T. On complex lines, the zero set of Φ is $N^5 \subset PT$.



Red lines are $\mathbb{C}P^1 \subset PT$ (or $\mathbb{C}^2 \subset T$). Lines in N^5 correspond to points in Minkowski space. When two lines intersect, corresponding points are light-like separated.

Conformal geometry of Minkowski space-time

The space of lines $\mathbb{C}P^1 \subset N^5$ gives a compactified version of Minkowski space-time The group $SU(2,2)$ preserves $\Phi = 0$ and acts on Minkowski space-time by conformal transformations. $SU(2,2) = Spin(4,2)$ is a double cover of the conformal group $SO(4,2)$.

The Penrose transform

Solutions of helicity $s > 0$ massless wave equations on Minkowski space-time correspond to elements of the sheaf cohomology group

$$H^1(PT^+, \mathcal{O}(-2s-2))$$

The conformal group acts on these spaces of solutions. For physicists twistor methods help understand the conformal symmetry of solutions of massless wave equations. For mathematicians (representation theorists) they provide interesting examples of infinite-dimensional unitary representations of the non-compact group $SU(2,2)$.

Other metric signatures

Twistor theory most naturally provides a complexified version of (compactified) Minkowski space-time. This is the Grassmanian $Gr_{2,4}(\mathbb{C})$ of all $\mathbb{C}^2 \subset T$ (or all $\mathbb{C}P^1 \subset PT$), which is 4-complex dimensional. The group $SL(4,\mathbb{C}) = Spin(6,\mathbb{C})$ acts linearly on T and on $Gr_{2,4}(\mathbb{C})$ or PT. There are three interesting 4-real dimensional subspaces of $Gr_{2,4}(\mathbb{C})$ with complexification $Gr_{2,4}(\mathbb{C})$:

- Compactified Minkowski space-time as described earlier. This is determined by the choice of Φ. The conformal group $Spin(4,2)$ is a real form of $SL(4,\mathbb{C}) = Spin(6,\mathbb{C})$.
- The real Grassmanian $Gr_{2,4}(\mathbb{R})$. This is determined by the standard real structure on T (or PT) and is acted on by the real form $Spin(3,3)$ of $Spin(6,\mathbb{C})$.
- The sphere S^4, acted upon by the conformal group $Spin(5,1)$, which is another real form of $Spin(6,\mathbb{C})$. This is the version of space-time with Euclidean signature metric that interests us.

Euclidean twistor theory

Euclidean signature space-time twistors are best understood using quaternions. One can identify $T = \mathbb{C}^4 = \mathbb{H}^2$ and use the fact that $S^4 = \mathbb{H}P^1$, quaternionic projective space. The conformal group $Spin(5,1) = SL(2,\mathbb{H})$ acts transitively on PT and S^4 through its linear action on $\mathbb{H}^2$.

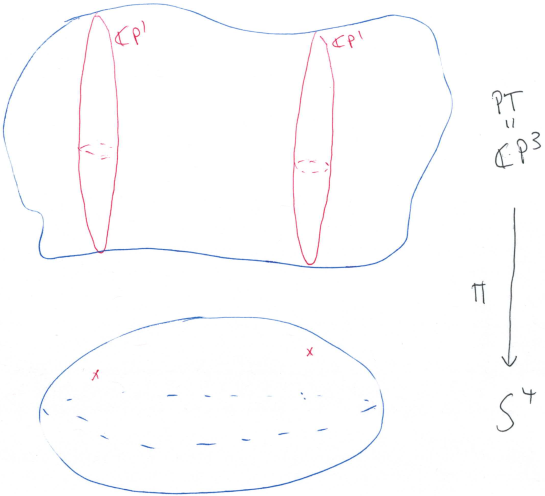
One has a fibration with fibers $\mathbb{C}P^1$

$$\begin{array}{ccc} \mathbb{C}P^1 & \longrightarrow & PT = \mathbb{C}P^3 \\ & & \Big\downarrow \pi \\ & & S^4 \end{array}$$

where the map π takes a complex line in $\mathbb{C}^4$ to the quaternionic line it generates.

This deserves a picture (compare to more complicated Minkowski picture).

Euclidean twistor fibration: a picture



Two interpretations of PT

PT is the projective spin bundle $P(S_R)$

The fiber at a point is the $\mathbb{C}P^1$ of projective S_R space.

PT is the bundle of complex structures on S^4

The $\mathbb{C}P^1 = S^2$ fiber above a point on S^4 can be identified with the possible choices of complex structure on the tangent space at the point.

These definitions generalize PT to give a twistor space for any Riemannian manifold in $d = 4$. If the metric is anti-self-dual, this twistor space is a complex manifold and allows study of the Riemannian geometry using holomorphic methods.
For a hyperkähler manifold M, this generalization of PT is the product space

$$M \times \mathbb{C}P^1$$

The twistor real structure on $\mathbb{C}P^3$

On a complex manifold such as $\mathbb{C}P^3$, one can ask about "real structures" which are anti-holomorphic maps

$$\rho : \mathbb{C}P^3 \to \mathbb{C}P^3$$

such that $\rho^2 = 1$.

One gets a real structure from conjugation of complex coordinates, but there is another one, the "twistor real structure" ρ_{tw}. If one identifies $\mathbb{C}^4$ and $\mathbb{H}^2$ with their corresponding $\mathbf{i}$, then multiplication by $\boldsymbol{j}$ is an anti-holomorphic map satisfying $\boldsymbol{j}^2 = -1$ on $\mathbb{C}^4$ and inducing an anti-holomorphic map ρ_{tw} with square 1 on $\mathbb{C}P^3$.

This ρ_{tw} is the structure needed to get Euclidean signature space time out of PT. The action of ρ_{tw} on PT has no fixed points, but it does have fixed $\mathbb{C}P^1$s, in fact a four-dimensional family of them parametrized by S^4 which fibers PT.

See previous picture.

The twistor $\mathbf{P}^1$

Each $\mathbb{C}P^1$ fiber comes with a real structure ρ_{tw} with no fixed points, identifying $\mathbb{C}P^1 = S^2$. This is the antipodal map.
Identifying $\mathbb{C}^2$ with the quaternion $z_1 + z_2\boldsymbol{j}$. One gets, in homogeneous coordinates $[z_1, z_2]$ or coordinate $z = z_1/z_2$

$$\rho_{tw}([z_1, z_2]) = [-\overline{z_2}, \overline{z_1}], \quad \rho_{tw}(z) = -1/\overline{z}$$

In a sense I won't try and make precise here, there are two "real forms" of $\mathbb{C}P^1$, something defined over $\mathbb{R}$ that becomes $\mathbb{C}P^1$ when you extend scalars to $\mathbb{C}$. The real structure on $\mathbb{C}P^1$ gives the action of the Galois group $Gal(\mathbb{C}/\mathbb{R}) = \mathbb{Z}/2\mathbb{Z}$. These are $\mathbb{R}P^1$ for the usual real structure, the twistor $\mathbf{P}^1$ for ρ_{tw}.
Another point of view on this is that there are two different 4d algebras over the reals that complexify to $M(2,\mathbb{C})$: $M(2,\mathbb{R})$ and $\mathbb{H}$.

An advertisement

Last fall I became fascinated by the fact that the twistor P^1 that describes a space-time point in Euclidean signature twistor theory also appears in new work by Fargues-Scholze on the Langlands program in number theory. They reformulate the local Langlands conjecture for each prime in terms of geometric Langlands on something called the Fargues-Fontaine curve. The Fargues-Fontaine curve at the infinite prime is the twistor P^1.

For more about this, see

`https://arxiv.org/abs/2104.05099`

Relating Euclidean and Minkowski

In a quantum field theory formulated on Minkowski space-time, there is no distinguished time direction, no need for such a thing to define states and operators. The situation is very different for a quantum field theory formulated on Euclidean space-time. To define physical states and operators one needs to pick an imaginary time direction, with asymmetry in $\pm$ imaginary time corresponding to the physical asymmetry in $\pm$ energy. In terms of symmetries, you need to break $SO(4)$ covariance by choosing a $\tau = 0$ hyperplane and using (Osterwalder-Schrader) reflection in that hyperplane. This will allow one to get from the $SO(4)$ covariant Euclidean Fock space theory to a physical Fock space theory with $SO(3,1)$ covariance.

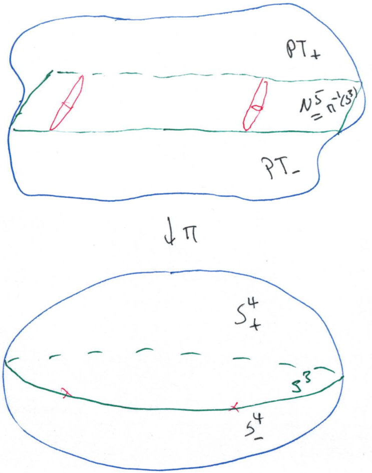
Minkowski and Euclidean QFT are very different

Minkowski	Euclidean
Positive energy condition: $\widehat{f}(E)$ supported on $E > 0$	$f(t) = \int_{-\infty}^{\infty} \widehat{f}(E)e^{iEt}dE$ is holomorphic on the upper half complex time plane ($\tau > 0$)
Field operators satisfy a wave equation	Field operators satisfy no equation of motion (always off-shell)
Field operators don't commute	Field operators always commute
Physical state space can be defined Lorentz covariantly (can specify $E > 0$ covariantly)	Defining physical state space requires breaking 4d rotational invariance (can't specify $\tau > 0$ without breaking $SO(4)$)
The Lorentz group $SO(3,1)$ acts on physical states and operators	The rotation group $SO(4)$ acts on Euclidean Fock space states and operators, but these are not physical states or operators

Euclidean twistor fibration and continuation to Minkowski

In twistor geometry the new structure needed on PT to get to Minkowski signature is a 5-dimensional hypersurface N^5 which splits it into two pieces. N^5 is the inverse image under π of an equator $S^3 \subset S^4$ that one can think of as the $\tau = 0$ subspace for some choice of imaginary time direction.
For different choices of imaginary time direction, you get different N^5s and different versions of Minkowski space-time as lines in N^5.
This deserves another picture.

Another picture, including distinguished imaginary time



Unification: general relativity as a gauge theory

There's a long history of attempts to treat Einstein's general relativity as a gauge theory, trying to emulate the success of the Yang-Mills gauge theory. One can formulate GR as a gauge theory, taking

- $G = SO(3,1)$ and the principal G-bundle of orthonormal frames on spacetime M.
- A connection ω (the spin-connection) with curvature Ω on this bundle
- A frame bundle comes with an $\mathbb{R}^4$-valued canonical 1-form e (the vierbeins).
- The Palatini action is

$$\int_M \epsilon_{ABCD} e^A \wedge e^B \wedge \Omega^{CD}(\omega)$$

Equations of motion: from varying ω, ω is torsion-free (Levi-Civita connection), from varying e, get the Einstein equations.

Euclidean signature general relativity

If we work in Euclidean signature spacetime, ω takes values in $\mathfrak{spin}(4) = \mathfrak{su}(2)_R \oplus \mathfrak{su}(2)_L$.

We can just use the $\mathfrak{su}(2)_R$ component ω_R, and its curvature Ω_R and still get the Einstein equations. One way to do this is to just replace Ω in the Palatini action by Ω_R. Both ω_R and Ω_R act on S_R spinors, not on S_L spinors. Remarkably, one can recover the Einstein equations just using ω_R, Ω_R.

Note that this doesn not work in Minkowski space-time, where ω takes values in $\mathfrak{so}(3,1) = \mathfrak{sl}(2,\mathbb{C})$.

Gravi-weak unification

There have been attempts to unify the weak interactions with gravity, using the chiral decomposition of the spin connection as above, with $SU(2)_R$ a space-time symmetry giving a gravity theory, and $SU(2)_L$ the internal symmetry of a Yang-Mills theory of the weak interactions. Our proposal is of this nature, but with the following different features:

- Take the Euclidean signature QFT theory as fundamental, with Minkowski signature physics to be found later by analytic continuation.
- Note that in Euclidean QFT one component of the vierbein is distinguished (the imaginary time direction).
- Use twistor geometry to get not just an $SU(2)_L$ internal symmetry but the full electroweak $SU(2)_L \times U(1)$ electroweak internal symmetry, with the imaginary time component of the vierbein behaving like a Higgs field.

Twistor unification: gravi-weak

If one works on the projective twistor space PT, one can get the idea of gravi-weak unification to work (in its Euclidean form):

- There is not just an $SU(2)$ internal symmetry, but also a $U(1)$, given by the complex structure specified by the point in the fiber. This complex structure picks out a $U(2) \subset SO(4)$, the complex structure preserving orthogonal transformations of the tangent space to the point on the base S^4. This is the electroweak $U(2)$ symmetry, to be gauged to get the standard electroweak gauge theory.
- If one lifts the choice of vector in the imaginary time direction up to PT, it transforms like the Higgs field: it is a vector in $\mathbb{C}^2$ (using the complex structure on the tangent space given by the point in the fiber). The $U(2)$ act on this $\mathbb{C}^2$ in the usual way. Each choice of Higgs field breaks the $U(2)$ down to a $U(1)$ subgroup, which will be the unbroken gauge symmetry of electromagnetism.

Twistor unification: QCD

Besides specifying a point on S^4 and a complex structure on its tangent space, a point in PT specifies a complex line $\mathbb{C} \subset \mathbb{C}^4$. The $U(1)$ discussed above is the group of phase transformations of that complex line. At the same time, the point in PT specifies a three-complex dimensional space, the quotient space $\mathbb{C}^4/\mathbb{C}$. Using the standard Hermitian form on $\mathbb{C}^4$, the group $SU(4)$ acts on $\mathbb{C}^4$ preserving this form.

Looking at this action as an action on the space of lines $PT = \mathbb{C}P^3$, the stabilizer of a point is the group $U(3)$. This includes the $U(1)$ which acts on the line, but also an $SU(3)$ that acts on the quotient.

Using the quaternion picture we've found that a choice of a point on S^4 gives a decomposition $\mathbb{H}^2 = \mathbb{H} \oplus \mathbb{H}$ and picks out an $Sp(1) \times Sp(1)$ subgroup of $Sp(2)$.

Using the complex picture, a point on PT gives a decomposition $\mathbb{C}^4 = \mathbb{C} \oplus \mathbb{C}^3$ and picks out a $U(3)$ subgroup of $SU(4)$. We thus have the right internal and spin rotation symmetries to gauge and get a unified theory.

A generation of matter fields

A generation of SM matter fields has exactly the transformation properties under the SM gauge groups as maps from $\mathbb{C}^4$ to itself, or

$$Hom(\mathbb{C} \oplus \mathbb{C}^3, S_R \oplus S_L) = (\mathbb{C} \oplus \mathbb{C}^3)^* \otimes (S_L \oplus S_R)$$

One could write this space as

$$(\mathbb{C}_{-1} \otimes \mathbb{C}^3_{\frac{1}{3}}) \otimes (\mathbb{C}^2_0 \oplus \mathbb{C}_{-1} \oplus \mathbb{C}_{+1})$$

which is

$$\mathbb{C}^2_{-1} \oplus \mathbb{C}_{-2} \oplus \mathbb{C}_0 \oplus (\mathbb{C}^3 \otimes \mathbb{C}^2)_{\frac{1}{3}} + \mathbb{C}^3_{-\frac{2}{3}} + \mathbb{C}^3_{\frac{4}{3}}$$

Here the subscripts are $U(1)$ weights (weak hypercharge), the $\mathbf{C}^2$ are the fundamental representation of $SU(2)_L$ and the $\mathbb{C}^3$ are the fundamental representation of $SU(3)$. For the first generation, the terms above correspond respectively to the fundamental particles

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, e_R, (\nu_e)_R, \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R$$

Problems and opportunities

In this proposal, the fundamental symmetries and degrees of freedom of GR and the SM are there, but in an unusual reorganized form. For instance, some degrees of freedom now live on points of PT which one can think of as light-rays, rather than on points of space-time. One needs to find a formalism on PT that corresponds to the usual Yang-Mills formalism on the base S^4. Need to use holomorphicity on the $\mathbb{C}P^1$ fibers to match degrees of freedom on S^4 and on PT. This requires a rethinking of the usual foundations of the theory. Resolving these questions may provide opportunities for addressing some long-standing problems (e.g. the renormalizability of the gravity theory). Work in progress...

Attractive aspects of this picture of fundamental physics

- Spinors are tautological objects (a point in space-time is a space of Weyl spinors), rather than complicated objects that must be separately introduced in the usual geometrical formalism.
- Analytic continuation between Minkowski and Euclidean space-time can be naturally performed in twistor geometry.
- Exactly the internal symmetries of the Standard Model occur.
- The intricate transformation properties of a generation of Standard Model fermions correspond to a simple construction.
- One gets a new chiral formulation of gravity, unified with the SM.
- Conformal symmetry is built into the picture in a fundamental way.
- Points in space time are described by the $p = \infty$ analog of the Fargues-Fontaine description of the "points" p of number theory.