Rethinking the Foundations of Physics: Unification

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Outline

The Standard Model

- Forces due to $U(1) \times SU(2) \times SU(3)$ gauge fields (generalizing EM theory)
- Matter is three generations of spin $\frac{1}{2}$ Fermi fields, specific $U(1) \times SU(2) \times SU(3)$ charges
- Scalar Higgs field breaks $U(1) \times SU(2)$ to $U(1)$, provides masses of weak gauge bosons and matter fermions

History: was basically in place April 1973

Experimental situation

ALL EXPERIMENTAL RESULTS AGREE PRECISELY WITH THEORETICAL PREDICTIONS

General Relativity

- Spacetime is a $3+1$ dimensional pseudo-Riemannian manifold (locally looks like Minkowski spacetime)
- Gravitational force is described by curvature of spacetime: Einstein-Hilbert action

History: was basically in place 1915

Experimental situation

ALL EXPERIMENTAL RESULTS AGREE PRECISELY WITH THEORETICAL PREDICTIONS

Questions unification should answer

Both theories are geometrical, largely determined by their symmetries. They leave some obvious questions unanswered:

- Why $U(1) \times SU(2) \times SU(3)$? Why the values of the three corresponding coupling constants?
- Why do matter particles have spin $\frac{1}{2}$ and the specific pattern of charges? Why 3 generations?
- Why the Higgs field and its potential? Why the Yukawa couplings to matter?

A technical problem: still no consistent non-perturbative definition of chiral gauge theories.

Problem that has gotten the most attention: conventional quantization of weakly coupled GR has renormalizability problems. No consistent non-perturbative definition of quantum gravity, either by itself or coupled to the SM.

Grand Unified Theories

History: Begins with Georgi-Glashow (January 1974)

Basic idea:

- Choose a larger Lie group G such that $U(1) \times SU(2) \times SU(3) \subset G$. Grand unified theory is a gauge theory with gauge group G (simplest examples $G = SU(5), SO(10)$). Get a relation between the three SM coupling constants.
- Find irreducible representations of G that give the SM charges when you restrict to the SM subgroup.
- Introduce new Higgs fields, with dynamics that breaks G to $U(1) \times SU(2) \times SU(3)$.

Initial enthusiasm: The new generators in G give gauge fields that will cause quarks to decay to leptons, so get predictions for proton decay.

Experimental situation

PROTONS DON'T DECAY. ZERO EVIDENCE FOR CHARACTERISTIC PHENOMENA PREDICTED BY GUT THEORIES

Supersymmetry

History: Earliest supersymmetric extensions of SM in December 1974
Basic idea:

- Note that vectors V have "square roots": spinors S, i.e. $V = S \otimes S$ (more later). Extend the Poincaré Lie algebra to a super-Lie algebra, by adding fermionic spinor generators Q such that $\{Q, Q\} \sim P$.
- Extend the SM to a theory that has same gauge group, larger space-time symmetry algebra. Can also do this for a GUT.

Initial enthusiasm: The new generators in the super-Lie algebra commute with SM generators, so get prediction of new "superpartners" of all known particles.

Experimental situation

THERE ARE NO SUPERPARTNERS. ZERO EVIDENCE FOR CHARACTERISTIC PHENOMENA PREDICTED BY SUSY THEORIES

Supergravity and Kaluza-Klein Theories

History: Earliest full supergravity March 1976, 11d Kaluza-Klein version April 1978.

Basic idea:

- Supergravity is gauged version of SUSY, giving an extension of GR, with a gravitino partner to graviton.
- Kaluza-Klein versions use more than 4 spacetime dimensions.

Initial enthusiasm: Theory of everything. April 1980 Hawking lecture on "Is the End in Sight for Theoretical Physics?"

Experimental situation

NO EXTRA DIMENSIONS, NO GRAVITINOS

Superstring theory

History: Earliest proposal that a superstring theory could describe gravity, Scherk-Schwarz May 1974. Explosion of interest September 1984 with Witten's first paper.

Basic idea:

- Replace point particle theory with theory of one-dimensional extended objects.
- Bring together GUTs, Supergravity and Kaluza-Klein. Consistency requires ten spacetime dimensions, specific GUT group G.

Initial enthusiasm: Theory of everything, huge influence of Witten.

Experimental situation

NO EVIDENCE FOR GUTS, FOR SUSY, FOR SUPERGRAVITY, FOR EXTRA DIMENSIONS OR FOR STRINGS

Generic problems of unification attempts

Attempts to go beyond the SM and GR and find a more fully unified theory have all had the same generic problems:

- They try to embed the SM and GR in a larger structure, but there is no experimental evidence at all for any component of such a structure.
- Whatever the advantages are of the larger structure, they are ruined by the necessity of coming up with an explanation of why one doesn't see any evidence of this structure. Elegance then turns to ugliness.
- The theory fails in a very conventional way: whatever new it predicts is not seen, so one must make the theory more and more complicated to explain away why one sees nothing. The endpoint is not an elegant predictive theory, but a complicated one that predicts nothing.

I've emphasized that the ideas involved all are about 50 years old. These are not new ideas that need more work. The inability of the community to give up on failed ideas has brought the subject of unification to intellectual collapse. Most serious theorists have stopped working on the idea as hopeless without new help from experiment.

A proposal

The past few years I've been working on a different approach to unification which seems promising. It assumes:

- Four dimensions is very special. Unification should use not extra dimensions, but the special properties of 4d, in particular the geometry of 4d spinors and twistors.
- Quantum field theories are well defined only in Euclidean signature, with an extra structure needed to "Wick rotate" to Minkowski signature.

The main new idea is to recognize that this "Wick rotation" changes the spinor geometry in a fundamental way. One of the $SU(2)$ factors of the 4d rotation group becomes an internal symmetry from the point of view of Minkowski spacetime. This provides a new sort of unification of internal and spacetime symmetry, using just known degrees of freedom.

Euclidean vs. Minkowski quantum field theory

Wick rotation is supposed to be analytic continuation from real time t to imaginary time $i\tau$. But:

- **Operator formalism** As a function of complex time $z = t + i\tau$, Heisenberg picture fields satisfy

$$\phi(z = t + i\tau, \mathbf{x}) = e^{\tau H}\phi(z = t, \mathbf{x})e^{-\tau H}$$

  Since H has a positive unbounded spectrum, there will be problems with this for $\tau \neq 0$. You can't Wick rotate field operators from real to imaginary time.

- **Path integral formalism** The path integral gives a potentially well-defined measure in Euclidean spacetime, something ill-defined in Minkowski spacetime. You can't Wick rotate path integral measures from imaginary time to real time.

Wightman and Schwinger functions

While one can't analytically continue theories (operators, states, measures) one can analytically continue between

- **Wightman functions** These are defined as vacuum expectation values of real time operators, e.g.

$$W(x,y) = \langle 0|\,\phi(x)\phi(y)\,|0\rangle$$

  They are distributions, not functions, and in general $W(x,y) \neq W(y,x)$ (operators don't commute).

- **Schwinger functions** These are functions $S(x,y)$, moments of probability measures (path integrals), indexed by imaginary time. They are symmetric ($S(x,y) = S(y,x)$). They are zero temperature limits of a finite-temperature statistical mechanical calculation where one takes imaginary time to have finite extent $\beta = \frac{1}{kT}$.

Reconstructing the real time theory

Given just the Schwinger functions, one can reconstruct the real time states and operators, but doing this requires breaking the $SO(4)$ invariance of the imaginary time theory, by choosing an imaginary time direction, and an operator Θ_{OS} which implements "Osterwalder-Schrader reflection" in imaginary time.

Note that the way spacetime symmetries work is different in real and imaginary time. The construction of operators and states in real time is $SO(3,1)$ equivariant, with no distinguished direction of time (just $\pm$ timelike cones).

Osterwalder-Schrader reconstruction is well-understood for scalar field theories, but what happens for spinors has always been much more mysterious. My proposal is that it is here that something unexpected happens, with a spacetime symmetry in the Euclidean QFT appearing as in internal symmetry in the Minkowski QFT.

The problem with spinors

The fundamental problem is that the properties of 4d spinors are quite different in Minkowski and Euclidean spacetime. I don't have time or space here to go through the spinor geometry. Some basic facts though are:

Euclidean spacetime: The double cover of the rotation group is $Spin(4) = SU(2)_L \times SU(2)_R$, with the geometry in some sense breaking up into separate "left-handed" and "right-handed" parts, with the two $SU(2)$ groups acting on independent $\mathbf{C}^2$ Weyl spinors S_L and S_R. Vectors are the tensor product

$$S_L \otimes S_R$$

Minkowski spacetime: $Spin(4)$ is replaces by $Spin(3,1) = SL(2, \mathbf{C})$. There is one kind of $\mathbf{C}^2$ Weyl spinor S, together with its complex conjugate $\overline{S}$. Minkowski spacetime vectors are the tensor product $\overline{S} \otimes S$.

Wick rotating spinors

Note that while the Dirac operator

$$\not{D} = \gamma^\mu \frac{\partial}{\partial x^\mu}$$

is usually written this way to make it look like it is Lorentz invariant, it is not a scalar, but transforms non-trivially as a vector.
If one complexifies spacetime, one gets a complex four-vector, which transforms under the complex rotation group
$Spin(4, \mathbf{C}) = SL(2, \mathbf{C})_L \times SL(2, \mathbf{C})_R$ as $S_L \otimes S_R$.

The usual story

S_L and S_R are holomorphic representations, Wick rotation is analytic continuation in the complex spacetime $S_L \otimes S_R$.

A different Wick rotation for spinors

A different story

Wick rotation requires an appropriate Θ_{OS} for spinor fields. This will use the distinguished imaginary time direction to provide a distinguished Clifford algebra element (γ_0), which interchanges S_L and S_R.
What gets Wick rotated to Minkowski space-time is NOT $S_L \otimes S_R$ but $S_R \otimes S_R$. This is why I use the slogan **"spacetime is right-handed"**.
$SU(2)_L$ acts trivially on this, becomes an internal symmetry.

Still trying to better understand this, details to follow....