Euclidean Twistor Unification and the Twistor P^1

Peter Woit

Columbia University
Mathematics Department

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Outline

Note: These slides at
`https://www.math.columbia.edu/~woit/twistorunification/algpartqm.pdf`
Papers with more detail for the topics of this talk at
`https://arxiv.org/abs/2104.05099` and
`https://arxiv.org/abs/2202.02657`

Four-dimensional geometry and 2x2 complex matrices

One can do four-dimensional complex geometry by identifying $\mathbb{C}^4$ with 2x2 complex matrices

$$(z_0, z_1, z_2, z_3) \leftrightarrow z = z_0 1 - i(z_1\sigma_1 + z_2\sigma_2 + z_3\sigma_3)$$

and defining

$$|z|^2 = \det z$$

Pairs $g_L, g_R \in SL(2, \mathbb{C}) \times SL(2, \mathbb{C}) = Spin(4, \mathbb{C})$ act preserving $|z|$ by

$$z \to g_L z g_R^{-1}$$

We are interested in "real forms" of this (real 4d subspaces that give above after complexification).

Real forms

Three real forms are

- $(2,2)$ signature inner product: $Spin(2,2) = SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$, using $g_L, g_R \in SL(2,\mathbb{R})$.
- $(3,1)$ signature inner product: $Spin(3,1) = SL(2,\mathbb{C})$, using $g_R = (g_L^\dagger)^{-1}$
  This is Minkowski space-time.
- $(4,0)$ signature inner product: $Spin(4,0) = SU(2) \times SU(2)$, using $g_L, g_R \in SU(2)$.
  This is Euclidean space-time.

Our interest will be in the Minkowski and Euclidean cases, together with the analytic continuation relating them.

Euclidean signature and quaternions

In Euclidean signature, can use quaternions instead of complex matrices

$$(x_0, x_1, x_2, x_3) \leftrightarrow x = x_0 1 + x_1 \boldsymbol{i} + x_2 \boldsymbol{j} + x_3 \boldsymbol{k}$$

with $|x|^2 = x\overline{x}$ and rotations given by pairs q_l, q_R of unit length quaternions.

$$x \to q_L x q_R^{-1}$$

Note that when we do this, we now have a conjugation operation (changing sign of $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$).

Spinor geometry

If one expresses four-dimensional vectors as 2x2 complex matrices, one can think of vectors as linear maps from one $\mathbb{C}^2$ (called the (half)-spinor space S_R) to another $\mathbb{C}^2$ (called the (half)-spinor space S_L). Corresponding to the action on vectors by

$$x \rightarrow g_L x g_R^{-1}$$

we have actions on S_R, S_L by

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_R \in S_R \rightarrow g_R \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_R \in S_R$$

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_L \in S_L \rightarrow g_L \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_L \in S_L$$

Note that

- In Euclidean space, g_R and g_L are independent $SU(2)$ matrices.
- In Minkowski space, $g_R \in SL(2,\mathbb{C})$ and g_L is determined by g_R $(= (g_R^{-1})^\dagger)$.

Twistor theory

Twistor geometry is a different way of thinking about the geometry of space-time, first proposed in 1967 by Penrose. It naturally provides a joint complexification of Minkowski and Euclidean space-time and a way to look at analytic continuation between them.

$4d$ conformal symmetry is most easily understood using twistors, especially if one works with the conformal compactification of space-time (S^4 instead of $\mathbb{R}^4$ in the Euclidean case).

Most discussions for physicists focus on the Minkowski version, we're more interested in the Euclidean version, together with what is needed to do analytic continuation from Euclidean to Minkowski.

Suggested references:

- *Twistor Geometry and Field Theory* by Ward and Wells.
- Any expository article about twistors by Penrose, or The Road to Reality chapter on twistors.

Twistor space

In twistor theory one takes as fundamental twistor space $T = \mathbb{C}^4$ (or its projective version $PT = \mathbb{C}P^3$, the complex lines in T).
Points of space-time will correspond to a $\mathbb{C}^2 \subset T$, tautologically giving the fiber S_R of the half-spinor bundle.
Such $\mathbb{C}^2$ correspond to $\mathbb{C}P^1$s in PT. Looking at all $\mathbb{C}^2 \subset T$, one gets the Grassmanian $Gr_{2,4}(\mathbb{C})$ which is 4-complex dimensional. Compactified Euclidean or Minkowski space-time (or $Gr_{2,4}(\mathbb{R})$ are 4-real dimensional subspaces of $Gr_{2,4}(\mathbb{C})$, or equivalently, 4-real dimensional families of $\mathbb{C}P^1 \subset PT = \mathbb{C}P^3$.
In the physical Minkowski space-time, the $\mathbb{C}P^1$ describing a space-time point corresponds to the sphere of directions of light rays one sees when one opens an eye.
The conformal groups are given by real forms $Spin(5,1) = SL(2,\mathbb{H})$ (Euclidean), $Spin(4,2) = SU(2,2)$ (Minkowski) and $Spin(3,3) = SL(4,\mathbb{R})$ ($Gr_{2,4}$(R)) of the group $SL(4,\mathbb{C}) = Spin(6,\mathbb{C})$ which acts linearly on T.

Euclidean twistor theory

We'll concentrate on Euclidean space-time twistors, which are best understood using quaternions. One can identify $T = \mathbb{C}^4 = \mathbb{H}^2$ and use the fact that $S^4 = \mathbb{H}P^1$, quaternionic projective space. The conformal group $Spin(5,1) = SL(2,\mathbb{H})$ acts transitively on PT and S^4 through its linear action on $\mathbb{H}^2$.

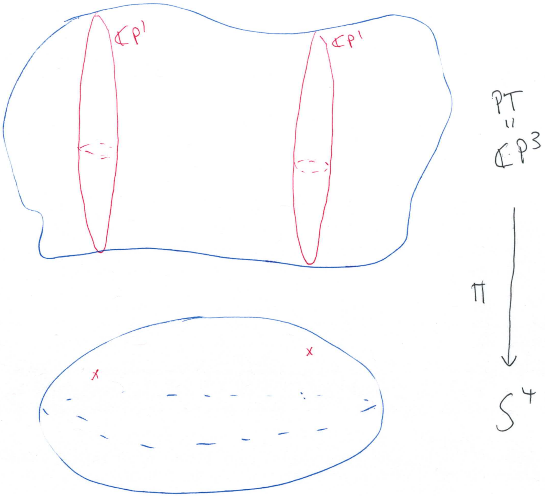
One has a fibration with fibers $\mathbb{C}P^1$

$$\begin{array}{ccc} \mathbb{C}P^1 & \longrightarrow & PT = \mathbb{C}P^3 \\ & & \big\downarrow \pi \\ & & S^4 \end{array}$$

where the map π takes a complex line in $\mathbb{C}^4$ to the quaternionic line it generates.

This deserves a picture:

Euclidean twistor fibration: a picture



Two interpretations of PT

PT is the projective spin bundle $P(S_R)$

The fiber at a point is the $\mathbb{C}P^1$ of projective S_R space.

PT is the bundle of complex structures on S^4

The $\mathbb{C}P^1 = S^2$ fiber above a point on S^4 can be identified with the possible choices of complex structure on the tangent space at the point.

These definitions generalize PT to give a twistor space for any Riemannian manifold in $d = 4$. If the metric is ASD, this twistor space is a complex manifold and allows study of the Riemannian geometry using holomorphic methods.

For a hyperkähler manifold M, this generalization of PT is the product space

$$M \times \mathbb{C}P^1$$

The twistor real structure on $\mathbb{C}P^3$

On a complex manifold, one can ask about "real structures" which are anti-holomorphic maps

$$\rho : \mathbb{C}P^3 \to \mathbb{C}P^3$$

such that $\rho^2 = 1$.

One gets a real structure from conjugation of complex coordinates, but there is another one, the "twistor real structure" ρ_{tw}. If one identifies $\mathbb{C}^4$ and $\mathbb{H}^2$ with their corresponding i, then multiplication by $\boldsymbol{j}$ is an anti-holomorphic map satisfying $\boldsymbol{j}^2 = -1$ on $\mathbb{C}^4$ and inducing an anti-holomorphic map ρ_{tw} with square 1 on $\mathbb{C}P^3$.

This ρ_{tw} is the structure needed to get Euclidean signature space time out of PT. The action of ρ_{tw} on PT has no fixed points, but it does have fixed $\mathbb{C}P^1$s, in fact a four-dimensional family of them parametrized by S^4 which fibers PT.

See previous picture.

The twistor P^1

Each $\mathbb{C}P^1$ fiber comes with a real structure ρ_{tw} with no fixed points, identifying $\mathbb{C}P^1 = S^2$. This is the antipodal map.
Identifying C^2 with the quaternion $z_1 + z_2\boldsymbol{j}$. One gets, in homogeneous coordinates $[z_1, z_2]$ or coordinate $z = z_1/z_2$

$$\rho_{tw}([z_1, z_2]) = [-\overline{z_2}, \overline{z_1}], \quad \rho_{tw}(z) = -1/\overline{z}$$

In a sense I won't try and make precise here, there are two "real forms" of $\mathbb{C}P^1$, something defined over $\mathbb{R}$ that becomes $\mathbb{C}P^1$ when you extend scalars to $\mathbb{C}$. The real structure on $\mathbb{C}P^1$ gives the action of the Galois group $Gal(\mathbb{C}/\mathbb{R}) = \mathbb{Z}/2\mathbb{Z}$. These are $\mathbb{R}P^1$ for the usual real structure, the twistor P^1 for ρ_{tw}.
Another point of view on this is that there are two different 4d algebras over the reals that complexify to $M(2, \mathbb{C})$: $M(2, \mathbb{R})$ and $\mathbb{H}$.

Analogs in number theory for all primes p

In number theory, for all primes p one has an analog of $\mathbb{R}$, the field of p-adic numbers $\mathbb{Q}_p$. The fields $\mathbb{Q}_p$ have a much more complicated set of field extensions than $\mathbb{R}$, but the quadratic field extensions K can be studied using the same structures as in the $\mathbb{R}$ case, equivalently:

- Two inequivalent Q_p forms of $M(2, K)$: $M(2, \mathbb{Q}_p)$ and an inequivalent quaternion algebra.
- Two Q_p forms of the projective line over K: the projective line over $\mathbb{Q}_p$ and a p-adic analog of the real twistor P^1.

There is a much more sophisticated analog in arithmetic geometry for all p of the twistor P^1, the Fargues-Fontaine curve FF_p. This is an analog of something studied by Carlos Simpson: the twistor P^1 with a chosen coordinate z and corresponding $\mathbb{C}^*$ action.

Recent developments in number theory

The Fargues-Fontaine curve that gives an analog for all p of the twistor P^1 has found a role in:

- Hodge theory: Simpson gave a reformulation of conventional Hodge theory in which $U(1)$ equivariant vector bundles over the twistor P^1 play the role of real Hodge structures. One can formulate p-adic Hodge theory in terms of $Gal(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$-equivariant vector bundles over FF_p.
- Local Langlands: Fargues and Scholze have shown that one can formulate the arithmetic local Langlands conjecture in terms of the geometric Langlands conjecture on FF_p.

The Fargues-Fontaine curve vs. the twistor $\mathbb{P}^1$

finite p	infinite p
$\mathbb{Q}_p$	$\mathbb{R}$
$\mathbb{C}_p$, completion of $\overline{\mathbb{Q}}_p$	$\mathbb{C}$
Fargues-Fontaine curve FF_p	$\mathbb{P}^1_{tw}$
pt. at ∞ given by $i : Spec\ \mathbb{C}_p \rightarrow FF_p$	$0, \infty \in \mathbb{CP}^1$, $\infty \in \mathbb{P}^1_{tw}$
$Gal(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ action on FF_p	$U(1)$ action on $\mathbb{P}^1_{tw}$
vector bundles on FF_p	vector bundles on $\mathbb{P}^1_{tw}$
vector bundles classified by fractions	vector bundles classified by half-integers
$Gal(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$-equivariant vector bundles on FF_p	$U(1)$-equivariant vector bundles on $\mathbb{P}^1_{tw}$
p-adic Hodge structure	Hodge structure
Fontaine ring $B^+_{dR} = \widehat{\mathcal{O}}_{FF_p,\infty}$	Power series ring $\mathbb{C}[[\lambda]] = \widehat{\mathcal{O}}_{\mathbb{P}^1_{tw},\infty}$
$\widehat{\mathbb{Z}}$-cover of FF_p	$\mathbb{CP}^1$ is $\mathbb{Z}/2\mathbb{Z}$-cover of $\mathbb{P}^1_{tw}$

Why Euclidean QFT?

Going back from number theory to physics, the philosophy we will pursue is that fundamental theory should be defined in Euclidean signature space-time, our observed physical space time is an analytic continuation. On reason is that QFT has inherent definitional problems in Minkowski signature that don't occur in Euclidean signature:

Non-perturbative (path integral) problems

Looking at the path integrals

$$\int F[\phi]e^{iS_M(\phi)}d\phi \text{ versus } \int F[\phi]e^{-S_E(\phi)}d\phi$$

If you do rigorous mathematics you can't make sense of the first, can make sense of the second (ask a mathematical physicist).
If you do numerical calculations, same thing (ask a lattice gauge theorist).

Euclidean QFT and the two-point function: momentum space

Every quantum field theory textbook explains that there's a problem even in free field QFT. Computing the two-point function involves taking the Fourier transform of

$$\frac{i}{\omega_{\mathrm{p}}^2 - E^2}$$

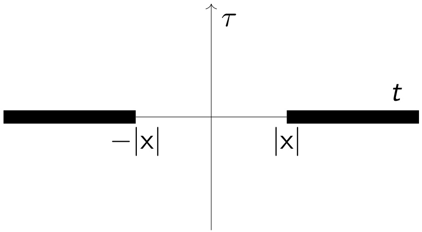
where

$$\omega_{\mathrm{p}}^2 = |\mathrm{p}|^2 + m^2$$

To do this you have to decide what to do about at the poles $E = \pm\omega_{\mathrm{p}}$. The physically sensible answer corresponds to analytically continuing the calculation from Euclidean space-time.

Euclidean QFT and the two-point function: position space

In position space, as a function of complex time, the two-point function is well-defined except along the real axis for time-like $(t, \mathbf{x})$. There it needs to be defined as a distribution given by a boundary value of a holomorphic function (a "hyperfunction"), as one takes a limit approaching the real axis.



Two-point functions are functions in Euclidean signature, hyperfunctions in Minkowski signature.

Minkowski and Euclidean QFT are very different

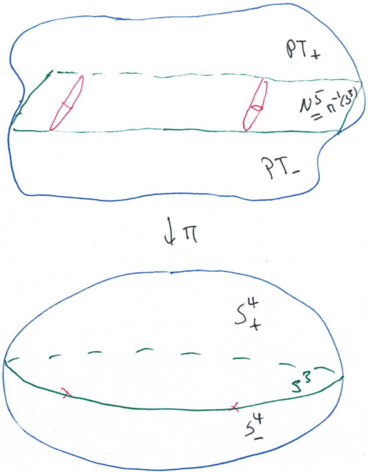
here

Minkowski	Euclidean
Positive energy condition: $\widehat{f}(E)$ supported on $E > 0$	$f(t) = \int_{-\infty}^{\infty} \widehat{f}(E)e^{iEt}dE$ is holomorphic on the upper half complex time plane ($\tau > 0$)
Field operators satisfy a wave equation	Field operators satisfy no equation of motion (always off-shell)
Field operators don't commute	Field operators always commute
Physical state space can be defined Lorentz covariantly (can specify $E > 0$ covariantly)	Defining physical state space requires breaking 4d rotational invariance (can't specify $\tau > 0$ without breaking $SO(4)$)
The Lorentz group $SO(3,1)$ acts on physical states and operators	The rotation group $SO(4)$ acts on Euclidean Fock space states and operators, but these are not physical states or operators

Relating Euclidean and Minkowski

To get from Euclidean signature space-time to Minkowski space-time, one must pick an imaginary time direction, with asymmetry in $\pm$ imaginary time corresponding to the physical asymmetry in $\pm$ energy. In terms of symmetries, you need to break $SO(4)$ covariance by choosing a $\tau = 0$ hyperplane and using (Osterwalder-Schrader) reflection in that hyperplane. This will allow one to get from the $SO(4)$ covariant Euclidean Fock space theory to a physical Fock space theory with $SO(3,1)$ covariance.
In twistor geometry the new structure needed on PT to get to Minkowski signature is a 5-dimensional hypersurface N^5 which splits it into two pieces.
Another picture:

Euclidean twistor fibration: distinguished imaginary time



Minkowski space twistors

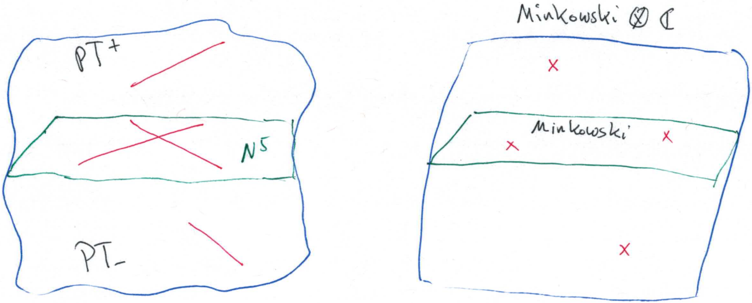
The subspace N^5 determines the Minkowski space-time geometry as follows. N^5 is the zero-set of a nondegenerate signature $(2,2)$ Hermitian form Φ on C^4. Minkowski space-time is the subspace of $\mathbb{C}^2 \subset \mathbb{C}^4$ on which $\Phi = 0$.

$\Phi = 0$ determines a real form $SU(2,2) = Spin(4,2)$ of $SL(4,\mathbb{C})$ that acts transitively on (compactified) Minkowski space. This is the conformal group, it also acts on solutions to massless wave equations. In the Euclidean case the conformal group real form was $SL(2,\mathbb{H})$.

The $\mathbb{C}P^1 = S^2$ in PT corresponding to a point in Minkowski space can be identified with the "celestial sphere" of light rays through that point. When two points are light-like separated, the corresponding $\mathbb{C}P^1$'s intersect.

Another picture:

Minkowski space-time twistors: a picture



General relativity as a gauge theory

There's a long history of attempts to treat Einstein's general relativity as a gauge theory, trying to emulate the success of the Yang-Mills gauge theory. One can formulate GR as a gauge theory, taking

- $G = SO(3,1)$ and the principal G-bundle of orthonormal frames on spacetime M.
- A connection ω (the spin-connection) with curvature Ω on this bundle
- A frame bundle comes with an $\mathbb{R}^4$-valued canonical 1-form e (the vierbeins).
- The Palatini action is

$$\int_M \epsilon_{ABCD} e^A \wedge e^B \wedge \Omega^{CD}(\omega)$$

Equations of motion: from varying ω, ω is torsion-free (Levi-Civita connection), from varying e, get the Einstein equations.

Euclidean Ashtekar variables

If we work in Euclidean signature spacetime, ω takes values in $\mathfrak{spin}(4) = \mathfrak{su}(2)_R \oplus \mathfrak{su}(2)_L$.

We can just use the $\mathfrak{su}(2)_R$ component ω_R, and its curvature Ω_R and still get the Einstein equations. One way to do this is to just replace Ω in the Palatini action by Ω_R. Both ω_R and Ω_R act on S_R spinors, not on S_L spinors. Remarkably, one can recover the Einstein equations just using ω_R, Ω_R.

In the Hamiltonian formalism a la Ashtekar, one notes that if one uses ω_R on a space-like hypersurface as configuration variable, the phase space is the same as in $SU(2)$ Yang-Mills theory, with the same sort of Gauss-law constraint from time-independent gauge transformations.

Instead of dynamics being determined by the Yang-Mills Hamiltonian, it is given by the constraints coming from diffeomorphism invariance of the Palatini action.

One usually tries to do this for $\mathfrak{so}(3,1)$ rather than $\mathfrak{spin}(4)$, this requires working with complexified variables.

Gravi-weak unification

There have been attempts to unify the weak interactions with gravity, using the chiral decomposition of the spin connection as above, with $SU(2)_R$ a space-time symmetry giving a gravity theory, and $SU(2)_L$ the internal symmetry of a Yang-Mills theory of the weak interactions. Our proposal is of this nature, but with the following different features:

- Take the Euclidean signature QFT theory as fundamental, with Minkowski signature physics to be found later by analytic continuation.
- Note that in Euclidean QFT one component of the vierbein is distinguished (the imaginary time direction).
- Use twistor geometry to get not just an $SU(2)_L$ internal symmetry but the full electroweak $SU(2)_L \times U(1)$ electroweak internal symmetry, with the imaginary time component of the vierbein behaving like a Higgs field.

Twistor unification: gravi-weak

If one works on the projective twistor space PT, one can get the idea of gravi-weak unification to work (in its Euclidean form):

- There is not just an $SU(2)$ internal symmetry, but also a $U(1)$, given by the complex structure specified by the point in the fiber. This complex structure picks out a $U(2) \subset SO(4)$, the complex structure preserving orthogonal transformations of the tangent space to the point on the base S^4. This is the electroweak $U(2)$ symmetry, to be gauged to get the standard electroweak gauge theory.
- If one lifts the choice of vector in the imaginary time direction up to PT, it transforms like the Higgs field: it is a vector in $\mathbb{C}^2$ (using the complex structure on the tangent space given by the point in the fiber). The $U(2)$ act on this $\mathbb{C}^2$ in the usual way. Each choice of Higgs field breaks the $U(2)$ down to a $U(1)$ subgroup, which will be the unbroken gauge symmetry of electromagnetism.

Twistor unification: QCD

Besides specifying a point on S^4 and a complex structure on its tangent space, a point in PT specifies a complex line $\mathbb{C} \subset \mathbb{C}^4$. The $U(1)$ discussed above is the group of phase transformations of that complex line. At the same time, the point in PT specifies a three-complex dimensional space, the quotient space $\mathbb{C}^4/\mathbb{C}$. Using the standard Hermitian form on $\mathbb{C}^4$, the group $SU(4)$ acts on $\mathbb{C}^4$ preserving this form.

Looking at this action as an action on the space of lines $PT = \mathbb{C}P^3$, the stabilizer of a point is the group $U(3)$. This includes the $U(1)$ which acts on the line, but also an $SU(3)$ that acts on the quotient.

Using the quaternion picture we've found that a choice of a point on S^4 gives a decomposition $\mathbb{H}^2 = \mathbb{H} \oplus \mathbb{H}$ and picks out an $Sp(1) \times Sp(1)$ subgroup of $Sp(2)$.

Using the complex picture, a point on PT gives a decomposition $\mathbb{C}^4 = \mathbb{C} \oplus \mathbb{C}^3$ and picks out a $U(3)$ subgroup of $SU(4)$. We thus have the right internal and spin rotation symmetries to gauge and get a unified theory.

A generation of matter fields

A generation of SM matter fields has exactly the transformation properties under the SM gauge groups as maps from $\mathbb{C}^4$ to itself, or

$$Hom(\mathbb{C}\oplus\mathbb{C}^3, S_R\oplus S_L)=(\mathbb{C}\oplus\mathbb{C}^3)^*\otimes(S_L\oplus S_R)$$

One could write this space as

$$(\mathbb{C}_{-1}\otimes\mathbb{C}^3_{\frac{1}{3}})\otimes(\mathbb{C}^2_0\oplus\mathbb{C}_{-1}\oplus\mathbb{C}_{+1})$$

which is

$$\mathbb{C}^2_{-1}\oplus\mathbb{C}_{-2}\oplus\mathbb{C}_0\oplus(\mathbb{C}^3\otimes\mathbb{C}^2)_{\frac{1}{3}}+\mathbb{C}^3_{-\frac{2}{3}}+\mathbb{C}^3_{\frac{4}{3}}$$

Here the subscripts are $U(1)$ weights (weak hypercharge), the C^2 are the fundamental representation of $SU(2)_L$ and the $\mathbb{C}^3$ are the fundamental representation of $SU(3)$. For the first generation, the terms above correspond respectively to the fundamental particles

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L, e_R, (\nu_e)_R, \begin{pmatrix}u\\ d\end{pmatrix}_L, u_R, d_R$$

Why three generations? Octonions?

While one generation fits into a very simple construction, no reason for 3 generations.

S^7 instead of $\mathbb{C}P^3$

Using quaternions and complex numbers, one has not fully exploited all the possible structures on the real eight-dimensional space T. In terms of unit vectors, S^7 carries several different kinds of geometry

$$S^7 = Spin(8)/Spin(7) = Spin(7)/G_2 = Spin(6)/SU(3) = Spin(5)/Sp(1)$$

In particular, we have used complex ($Spin(6) = SU(4)$) and quaternionic ($Spin(5) = Sp(2)$) aspects of the geometry, but not the octonionic aspects that appear in $S^7 = Spin(7)/G_2$.

We have gotten a lot of mileage out of thinking of $\mathbb{C}^2$ as $\mathbb{H}$ and from working with $\mathbb{H}^2 = T$. What about thinking of $\mathbb{H}^2 = \mathbb{O}$?

Gauge theory on PT

In this proposal, there's a profound reorganization of fundamental degrees of freedom. They now live on points of PT which one can think of as light-rays, rather than on points of space-time. Mathematically, one needs to find a formalism on PT that corresponds to the usual Yang-Mills formalism on the base S^4. Need to use holomorphicity on the $\mathbb{C}P^1$ fibers to match degrees of freedom on S^4 and on PT. The Penrose-Ward correspondence does this for anti-self-dual connections.

Similarly need to match the Dirac equation on S^4 and equations on PT. For bundles on the base with ASD connections, this is done by the Penrose-Ward correspondence, but the $U(1)$ and $SU(3)$ bundles are on PT, vary on a fiber.

True gravi-weak unification?

Have mostly just rewritten the usual electroweak and GR theory. One difference though is that one component of the vierbein is now the Higgs, which has the electroweak dynamics. Does this change the usual problems about renormalizability, higher order terms in the curvature, etc?

Attractive aspects of this picture of fundamental physics

- Spinors are tautological objects (a point in space-time is a space of Weyl spinors), rather than complicated objects that must be separately introduced in the usual geometrical formalism.
- Analytic continuation between Minkowski and Euclidean space-time can be naturally performed in twistor geometry.
- Exactly the internal symmetries of the Standard Model occur.
- The intricate transformation properties of a generation of Standard Model fermions correspond to a simple construction.
- One gets a new chiral formulation of gravity, unified with the SM.
- Conformal symmetry is built into the picture in a fundamental way.
- Points in space time are described by the $p = \infty$ analog of the Fargues-Fontaine description of the "points" p of number theory.