Unified Theories of Physics: their illustrious past, peculiar present, and uncertain future

Peter Woit

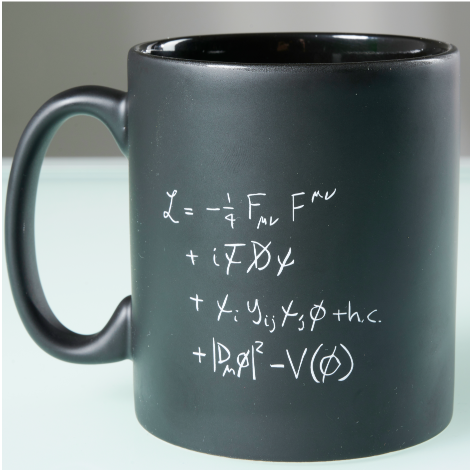
Mathematics Department
Columbia University

Oxford
September 27, 2022

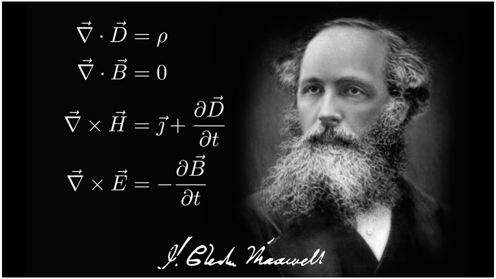
Unified theories in physics

You can purchase the mug on the right at the CERN gift shop. It summarizes our best unified theory of physics, but this equation is unchanged since 1973.

In this talk, I'll try and explain how this theory developed, and how we ended up in a peculiar static state for the past 50 years. I'll end with some speculation about how we might get out of this state and do better.

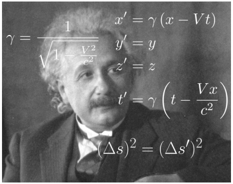


Electromagnetism: Maxwell's equations (1860s)



First line on the mug is: $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(|\mathbf{B}|^2 - |\mathbf{E}|^2)$
This gives the Maxwell equations above (by minimizing integral of the Lagrangian).
Maxwell's theory (together with Newton's mechanics) provided a unified theory describing very different physical phenomena (electricity, magnetism, light) in terms of a simple set of equations.

Special relativity: Einstein (1905)



We can think of space and time together as a 4-dimensional space-time with coordinates x, y, z, t. The Lagrangian $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is invariant under rotations (preserving length l, where $l^2 = x^2 + y^2 + z^2$), but also transformations of 4-d space-time (Lorentz transformations) that mix time and space, preserving (note the minus sign)

$$s^2 = x^2 + y^2 + z^2 - t^2$$

Einstein (1905): Not just electromagnetism, but all laws of physics should have this space-time symmetry. He showed how to modify Newton's mechanics to a "relativistic mechanics" with this property.

Quantum electrodynamics and the Dirac equation (1928)

Heisenberg/Schrödinger (1925-28) Quantum reinterpretation of Maxwell's theory. Photons are massless quanta of the $\mathbf{E}, \mathbf{B}$ fields.

Dirac equation: 1928

Dirac wrote down the equation now on his memorial at Westminster. This is the equation you get from the second line of the Lagrangian on the mug:

$$i\overline{\psi}\not{D}\psi$$

All matter particles are quanta of the field ψ.

Space-time and internal symmetry groups

Space-time symmetries

Rotations of 3-dimensional space form a "group" called O(3) (O for "orthogonal"). This is a group of symmetries (Lagrangian is invariant) for all fundamental physical theories.
Lorentz transformations of 4-dimensional space-time form a group called O(3,1). Our current fundamental theories have this symmetry.
Spinors: O(3) and O(3,1) act on Dirac's ψ field in a very subtle way, not as transformations of 3 or 4 real coordinates, but as transformations of 2 complex "spinor" coordinates.

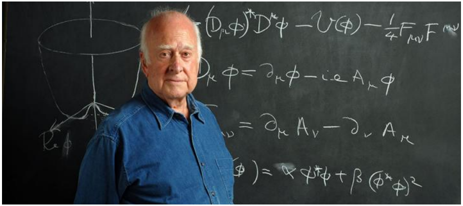
Internal symmetries

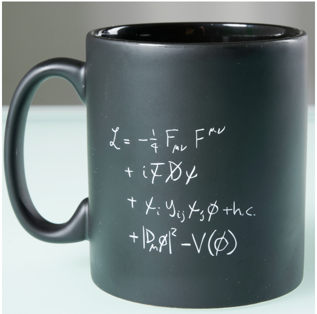
Quantum electrodynamics has another kind of symmetry, which doesn't act on space-time, but acts by multiplying fields by unit length complex numbers (which form the group called U(1)). One can do this independently at each space-time point, this is called a "gauge symmetry".

The Anderson-Higgs mechanism (1962-64)

Anderson (1962): If you study how quantum electromagnetic fields behave not in a vacuum, but in a superconducting medium, the photon become massive. He suggested the same mechanism should apply to some relativistic quantum theories in the vacuum.

Higgs and others (1964):
Introduce a complex scalar field ϕ
with the right potential energy,
and get such a theory.





Yang-Mills theory (1954)

Yang-Mills (1954): You can generalize the $U(1)$ internal gauge symmetry to larger groups, of higher dimension than $U(1)$ (which has dimension 1). Yang and Mills did this for a group called $SU(2)$, which is a group of two-by-two complex matrices. The group has three dimensions, and this implies that you have three analogs of the photon. The Lagrangian changes by

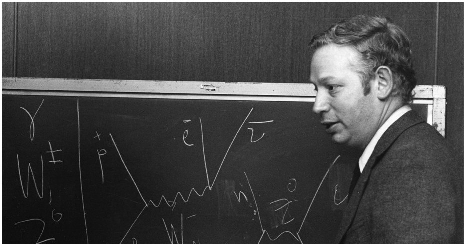
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \rightarrow \sum_{a=1}^{3} -\frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu}$$

The 2 by 2 SU(2) matrices act on the fields by multiplication, with the fields now doubled to allow this action. The matter Lagrangian changes by

$$i\overline{\psi}\not{D}\psi \rightarrow \sum_{j=1}^{2} i\overline{\psi}^{j}\not{D}\psi^{j}$$

For coffee cup notational simplicity, we'll make these kinds of sums implicit.

Electroweak unification (1967)

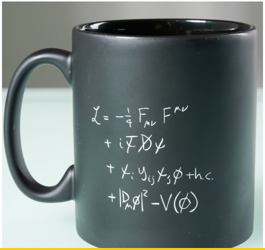


Weinberg (1967): Combining Yang-Mills and Anderson-Higgs, expand the QED internal symmetry from $U(1)$ to $U(1) \times SU(2)$ and introduce a scalar field to implement the Anderson-Higgs mechanism. This predicts three new massive versions of the photon (the W^+, W^- and Z^0), which are responsible for the weak interactions. Using notational simplification, the Lagrangian is just the one on the coffee cup.

QCD and the Standard Model (1973)

Gross-Wilczek, Politzer (1973): Remarkably, one can describe the strong interactions by using a Yang-Mills theory with three-by-three complex matrices and a group called SU(3) (no Higgs mechanism needed). A crucial role is played by the fact that for groups like $SU(2)$ and $SU(3)$ the analogs of the Maxwell equations are non-linear equations, with much more complicated behavior.

The Standard Model was now in place, with O(3,1) space-time symmetry and $U(1) \times SU(2) \times SU(3)$ internal gauge symmetry. With some notational simplification, the Lagrangian is just that on the coffee mug:



Some history since 1973

- 1974-5: Discovery of the charmed quark and tau lepton (more copies of known matter particles)
- 1977: Discovery of the bottom quark (as expected to fit with tau lepton).
- 1983: Discovery of W/Z bosons (as predicted)
- 1995: Discovery of the top quark (as expected to fit with bottom quark)
- 1998: Discovery of non-zero neutrino masses (consistent with coffee cup version of the SM)
- 2012: Discovery of the Higgs at the LHC (as predicted)

At our level of notational simplification, all of these vindicate and none of these change the Lagrangian on the coffee cup.

Our peculiar present

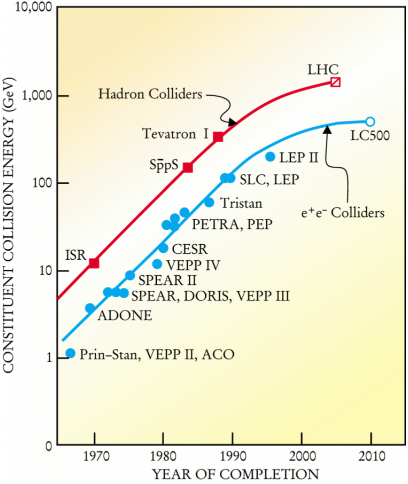
Since 1973 all experimental data we have been able to gather is consistent with the Standard Model. In particular, all the LHC data about the Higgs and its properties fits perfectly. But the Standard Model does not appear to be a final theory: there are questions it does not answer which we would like answers for, such as:

Open questions

- Why these particles, forces (e.g. why $U(1) \times SU(2) \times SU(3)$)?
- Why these parameters? Buried in our notational simplification are twenty-some parameters to be put in by hand. Most are in the matrix Y_{ij} in the coffee cup formula
- What about gravity? Unifying gravity with the SM remains an open problem.

We've run out of energy

To look for hints about how to proceed, one would like to do higher energy experiments, but:



The "LC500" $e^+ - e^-$ collider was not built, probably still at least 20 years off.
A p-p collider at significantly higher energy than the LHC: extremely expensive and not in my lifetime.

Technological limits at the high energy frontier

p-p colliders

Energy $\propto$ (radius)(magnetic field)
To double energy need to double circumference or double magnetic fields.
LHC circumference: 27 km
LHC magnets: 8 Tesla
Best magnets now: 16 Tesla

$\mathbf{e}^+ - \mathbf{e}^-$ colliders

- Circular colliders:
  Synchrotron radiation losses $\propto (Energy)^4/(radius)$
  LEP (209 GeV) power consumption = 40% city of Geneva
- Linear colliders:
  For given acceleration technology: Energy $\propto$ length
  Large power demand since beam dumped after acceleration, not stored.

Non-energy frontier experimental directions

One can do non-high energy experiments and look for non-SM physics:

- Neutrino physics
- Precision measurements
- Dark matter: astrophysics
- Cosmology

Only for dark matter does one see things arguably inconsistent with the SM, but this is not completely clear. A huge amount of current experimental and theoretical activity is aimed at the dark matter issue.

Quantum gravity

Experimental study of quantum gravity seems out of reach. Naive estimates say energy scale is 10^{16} times higher than what we can study with the LHC.

Enlarging fundamental symmetries: GUTs

Immediately after 1973, theorists started trying to build newer theories that would have more internal symmetry than the standard model:

Grand Unified Theories (GUTs), 1974

Extend internal symmetry to a larger group which includes $U(1) \times SU(2) \times SU(3)$ as subgroup. An example is the group $SU(5)$ built out of five by five complex matrices.

Problems

- New symmetry generators imply interactions that allow quarks to change into leptons. Protons then can decay, but experiments designed to look for such rare decays have seen nothing.
- These larger symmetry groups don't answer the question "why do we see $U(1) \times SU(2) \times SU(3)$?". You have to introduce additional complicated Higgs fields to explain this and choose their energy scales high enough to explain why none of this is observable.

Enlarging fundamental symmetries: SUSY

Another direction was to try and extend space-time symmetry to something larger

Supersymmetry (SUSY), 1977

Extend the group of space-time translations and Lorentz transformations ($O(3,1)$) to a larger "super"-group (allowing anticommuting variables).

Problems

- New symmetry generators imply "super-partner" states for all known elementary particles, but these have not been seen. LHC results conclusively negative.
- Since you don't see super-partners of the same mass as known particles, you need to introduce complex new physics to explain this, then make the energy scales so high that nothing is observable at LHC energies or below.

Supergravity (1976) unification

Can use SUSY as a gauge symmetry, and get "super-gravity" GUT theories incorporating gravity.
Hawking (1980): Inaugural lecture about this, title

"*Is the End in Sight for Theoretical Physics?*"

In principle this could give a fully unified theory including the SM and gravity. It has been the main model for attempts at unification since 1980.
Problems:

- Many new GUT and SUSY elements added to the SM, making theory more complicated. Zero experimental evidence for any of them.
- No explanation of any of the open questions about the SM.
- Internal consistency of gravity quantization unclear.

Superstring theory unification (1974)

Superstrings (1974): Take as fundamental not quantized particles, but quantized strings, in a supersymmetric version, the "superstring".

Problems:

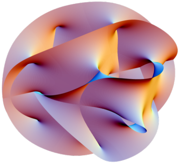
- Consistency requires 10 space-time dimensions.
- Vastly more complicated theory, only understood in the approximation of small string interactions. No known full theory.

From 1974-1984 very few people were interested in the idea.

1984: First superstring revolution

People had become discouraged by problems of supergravity unification, maybe this was the answer. Most influential theorist (Edward Witten) became enthusiastic and started working on this, many new developments.

String theory unification, the vision (1984-5)



The vision (1984-5)

Take 10d superstring as fundamental, compactify 6 dimensions using a special type of 6d space (Calabi-Yau), too small to be observable.
Get effective supergravity theory in 4d at low energy (Hawking's vision), unified theory of SM + quantum gravity.

String unification: problems with the 1984-85 vision

Calabi-Yaus come in parametrized families, need to not just fix a family, but also many "moduli" paramenters.

- Collaboration with mathematicians: more and more families of Calabi-Yaus. Currently unknown if the number of families is finite.
- Better understanding of the theory: more and more possibilities for dealing with extra 6 dimensions (e.g. branes). More and more possible "string vacua" (currently $10^{272,000}$ for each family).

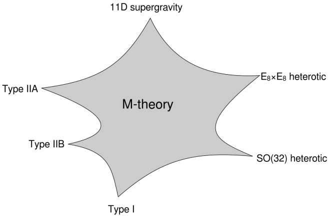
Research has steadily moved in the wrong direction, away from the vision

Better understanding the theory just keeps making the problem worse. More and more possible "approximate string vacua"

Fundamental problem

It appears that you can get just about any low energy physics you want, depending what you do with the extra dimensions. No predictions about observable physics.

What is string theory?



1995: Second superstring revolution

Conjectural full theory: M-theory

1997: AdS/CFT

New ideas about strongly coupled string theory in specific case, but no help with the "too many string vacua" problem

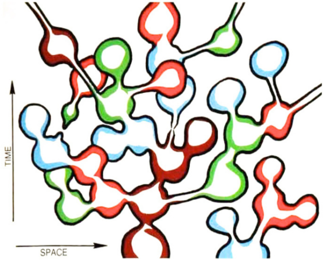
Current situation: "string theory" is not a theory, but a conjecture there is a theory

Typical summary talk by David Gross, Strings 20XX. "The big open questions are: What is string theory? What are the underlying symmetries of string theory?"

Fallout from string unification failure: the Multiverse

Where string theory unification vision has ended up

- Conjectured features of string theory imply if one "string vacuum" is consistent, so are an exponentially large number of them
- Can get essentially any low energy physics by choice of "string vacuum"
- Inflationary cosmology is invoked to create multiple universes and populate the possible "string vacua".
- No testable explanations. Nothing more than an elaborate excuse for a failed theory.



The end of the road for unification in physics?

Two influential arguments for giving up:

It's too hard

The SM is a victim of it's own success, it's too good. 50 years of failure to do better means the problem is too hard. Best to go do something else, the problem will have to wait for some future generation that gets new clues from experiment. Also, best to not discuss this failure publicly.

The multiverse vindicates string theory

String theory is not a failure, it predicts a multiverse which shows that we have to give up. String theory should become accepted science, as "our best unified theory", although there is no way to ever test it.

I've spent a lot of effort arguing against the second argument, without much success. Its influence is dangerous, discrediting a central part of science.

For more..

The discussion here has very oversimplified a complex story. For more details, see my blog (Not Even Wrong) and these three books:

Not Even Wrong (Peter Woit), 2006
The Trouble With Physics (Lee Smolin), 2006
Lost in Math (Sabine Hossenfelder), 2018

For a much more positive take on string theory, try
Why String Theory? (Joseph Conlon), 2016

A way forward?

The conventional point of view for decades has been that the Standard Model is just an effective low energy limit of some quite different fundamental theory valid at much higher energies where quantum gravitational effects become important, with string theory the leading contender. My own point of view has been that the dramatic agreement of the SM with experimental results at all accessible energies means one should take seriously the possibility that it is a fundamental theory valid to arbitrarily high energies.

If we are dealing with a truly fundamental theory, history has shown that such a theory will have deep connections to fundamental mathematics. This is already known to be true: the mathematics behind the SM is remarkably deep. So, can one make progress by better understanding the mathematical structure of the SM itself, finding structures that integrate better with the classical geometrical theory of gravity?

Some new ideas about unification

During the past two years I've become very enthusiastic about some new ideas concerning unification. Will try and say a little bit about this, but it is rather technical, would need a full hour to explain much.

Warning label

While I'm enthusiastic, I've so far gotten only a little interest from the theoretical physics community. This is not something vetted and accepted by the wider community.

References (see my website)

- arXiv preprint: https://arxiv.org/abs/2104.05099
- Talks at Brown (https://www.math.columbia.edu/~woit/twistorunification/brown9-23-21.pdf) and University of Texas at Dallas (https://www.math.columbia.edu/~woit/utdallas.pdf)
- Another preprint in the works...

Geometry of spinors and twistors

The most subtle and poorly understood part of the SM: spinor geometry

Spinors

As mentioned before, matter fields in the SM are given by two complex numbers ($\mathbf{C}^2$) at each point, which transform under rotations as "spinors". The geometry of these spinors is more fundamental than the usual vectors: you can make vectors out of spinors, but not vice-versa.

A wonderful idea (1967) of Penrose's is that of a "twistor". Oxford has been the main center for research on this topic.

Twistors

One can consider a "twistor" space given by four complex numbers ($\mathbf{C}^4$). The points of space-time are exactly the $\mathbf{C}^2$ lying inside this $\mathbf{C}^4$. The explanation for spinors is tautological: a point in space-time is a spinor space.

Analytic continuation in time

Recall that special relativity says space-time geometry uses as length the Minkowski version

$$s^2 = x^2 + y^2 + z^2 - t^2$$

One can make this the standard Euclidean notion of length in four dimensions by working instead with "imaginary time" $\tau = it$. It turns out that when we do calculations in quantum field theory we have to do this: the theory in terms of τ is well-defined, that in terms of t isn't. The usual prejudice is that this is just a mathematical trick of no fundamental significance.

Something that has always been confusing: spinor geometry in Euclidean space-time is very different than in Minkowski space time. Changing to imaginary time, you change the nature of the matter fields.

Is the $SU(2)$ internal symmetry of the SM a space-time symmetry in Euclidean space-time?

In the work that I have been pursuing, the way spinor geometry changes going from real time to imaginary time implies that one can think of four-dimensional rotations in imaginary time as corresponding to spatial rotations (which don't care which version of time you use) and the internal $SU(2)$ transformations of the electroweak part of the SM.

For many years I thought this kind of idea could not possibly work. During the past couple years I've become convinced that it does work. There is much to do to work out the details of this and see if one can get a new unified point of view on the symmetries of SM and of gravity theories. Such a point of view might point to a way forward for unifying these theories in a new way, resolving the usual difficulties.

Thanks for your attention!