Dear Prof. Woit,

You have recently said here that "The problem for Joshi is that Mochizuki strongly disagrees with him," thereby implying to your readers that my work can be dismissed. Mochizuki's comments you refer to are from March 2024 and the issue raised therein has been fixed. Perhaps you are unaware, but the current stable version of my papers were sent to Mochizuki and Scholze for comments in October 2024, and these versions were made available on the arxiv in February 2025. No further objections (since October 2024) have been made to me privately or publicly. You assert without evidence or citing any names that "experts do not think, $\cdots$" i.e. experts are whispering in your ears, but they are unwilling to bring any mathematical arguments to the table. *Why?* Is it not in the interest of the mathematical public?

The fact of the matter is that there are many uncomfortable issues which my work has uncovered. Notably, the report which has been championed by many arithmetic geometers falls apart under a detailed mathematical scrutiny. That some arithmetic geometers continue to use it to whisper to you what you want to hear, is deeply troubling. I am interested in mathematical truths, and I have independently established a canonical Arithmetic Teichmüller Theory which underpins Mochizuki's *abc*-conjecture claims (readers will find the FAQ [Joshi, 2025a] useful). The situation for Mochizuki is equally problematic, he too has championed his group of experts, who were also unable to provide any transparent articulation of his theory (and proofs), and he dismissed everyone else, especially his critics, as irrelevant. As can be seen from the missing citation to my work in [Boyd, 2025], Mochizuki is equally anxious to have my work dismissed (after all his papers are in print) and at this juncture this is simply to avoid sharing any credit. So, this unfortunate episode continues.

My goal has always been to precisely pin down Mochizuki's mathematical claims, and I have provided an original and independent approach for verification of his claims (notably, a canonical Arithmetic Teichmüller Theory does exist), using more familiar mathematics which makes my work very hard to refute (and because of clear enunciation and clarity of the proofs, my papers will be fully verifiable by Lean). At this point, this is a human drama and not a mathematical one.

I urge you to please stop projecting outdated and out of context statements about my work to your readers, or bring precise mathematical evidence to the conversation. Because instead of helping us arrive at a resolution of this issue, discrediting my work encourages some others to subsume my ideas as their own. As a professional mathematician, I will stand corrected if mathematical evidence emerges which proves me wrong–any such evidence will also apply equally to Mochizuki's claims.

Now, I am writing this to bring some factual foundations to various assertions made about [Mochizuki, 2021] in [Boyd, 2025].

§1. Here are some facts which should be kept in mind (also see FAQ [Joshi, 2025a]):

(1) The first reference to universes occurs in *Mochizuki's Key Principle of Inter-Universality* [Mochizuki, 2021, §I3, Pages 24-26] which, according to Mochizuki, is the foundational principle underlying Mochizuki's Theory. Except for a passing reference on [Mochizuki, 2021, Page 39], the term universe does not occur again in [Mochizuki, 2021, Pages 40-700]. The next reference occurs *after* the proof of the *abc*-conjecture is completed, on [Mochizuki, 2021, §3, Page 701] where Mochizuki initiates the meta-mathematical discussion which was the subject of Mr. Boyd's conversation with Mochizuki.

(2) Likewise, the phrase $\in$-loop does not occur anywhere until [Mochizuki, 2021, §3, Page 701].

(3) The term universe or $\in$-loop never occurs in [Scholze and Stix, 2018] as claimed in [Boyd, 2025].

Notably, neither universes nor $\in$-loops occur as mathematical devices in any of the main proofs

of Mochizuki's papers and do not occur in the proof of the *abc*-conjecture *at all*. Moreover, the objections raised in [Scholze and Stix, 2018] do not rest on the issue of universes or $\in$-loops. [See Questions 11, 16, 30–34 the (FAQ) [Joshi, 2025a].]

§2. So why is there a discussion of universes and $\in$-loops in [Mochizuki, 2021]? Before getting to the arithmetic context, it will be useful to understand the issue in the context of Riemann surfaces. Let X_1, X_2 be two compact Riemann surfaces of genus $g \geq 1$. Suppose further that X_1, X_2 are biholomorphic. Suppose we want to somehow assert that X_1 and X_2 are distinct i.e. non-isomorphic. This is impossible to do in the holomorphic category as X_1, X_2 are biholomorphic, and any attempt to declare X_1, X_2 to be distinct in the holomorphic category necessarily leads to contradictions. That is, what we want to do cannot be done in the language of holomorphic mappings.

Classical Teichmüller Theory resolves this dilemma by introducing quasi-conformal mappings. These mappings are not necessarily holomorphic (and therefore need not be algebraic). The key point is that the problem which we were unable to solve in the holomorphic category can now be solved in this bigger category of quasi-conformal mappings. *This is the picture Mochizuki wants to imitate in the arithmetic case.*

§3. Now how does one do this in the arithmetic case? Firstly, Mochizuki does not provide a satisfactory resolution of this issue in his papers. I have provided a detailed approach to this in my papers. Mathematically minded readers will find this FAQ [Joshi, 2025a] accessible, especially see Questions 6–13, 18–22, 24, 35.

§4. Note that [Boyd, 2025] makes no mention of Mochizuki's Key Principle of Inter-Universality (cited in §1 above (or see answer to [Joshi, 2025a, Question 31])) or even of classical Teichmüller Theory. Mochizuki's Teichmüller Theory claim cannot be understood without understanding this key principle. So *strictly speaking*, [Boyd, 2025] is of little mathematical relevance to [Mochizuki, 2021].

The said principle was a key point of my discussions with Scholze in [May-June 2024] as is stated in [Joshi, June, 2024]. As stated there, we both agreed that Mochizuki does not provide any mathematical way to distinguish between schematically isomorphic objects (i.e. the arithmetic analog of the situation described in §1), but as discussed in [Joshi, 2025a], arithmetic analog exists. For example, from the point of view of [Joshi, 2021], suitable Fargues-Fontaine curves are Arithmetic Teichmüller Spaces for $X = \mathrm{Spec}(\mathbb{Q}_p)$.

My work shows that there is a unique (arithmetic) Teichmüller Theory, whose existence is independent of the validity of the proof of the *abc*-conjecture, and which can be constructed using the said Principle. This theory underpins Mochizuki's *abc*-conjecture proof and his IUTT claims.

§5. To clearly summarize the situation regarding [Boyd, 2025] for your readers:

(1) As noted in §4, [Boyd, 2025] does not discuss Mochizuki's Key Principle of Inter-Universality and because of this Mr. Boyd's article has no real relevance for any discussion of [Mochizuki, 2021]. My work establishes the precise version of the unique theory which arises from this principle and which underlies Mochizuki's claims.

(2) [Boyd, 2025] attempts to rewind the clock back to 2018. It does not address the core issues (regarding [Mochizuki, 2021] and [Scholze and Stix, 2018]) which have already been highlighted ([Joshi, 2025b], [Joshi, June, 2024]) and effectively dealt with in my work done during 2019–2025.

(3) The conclusions which [Boyd, 2025] offers regarding [Mochizuki, 2021] are mathematically incorrect in the very specific context of elliptic curves relevant to Mochizuki's papers.

(4) A full and transparent discussion of other mathematical devices, such as prime-strips, which [Boyd, 2025] alludes to, can be found in [Joshi, 2024, §8]. A short accessible FAQ is found in [Joshi, 2025a].

(5) [Boyd, 2025] also refers to 'Arithmetic Teichmüller Theory' but makes no references (or gives credit) to my work on Arithmetic Teichmüller Theory (and my construction of Arithmetic Teichmüller Spaces which such a theory entails).

(6) I sent Mr. Boyd an email alerting him to my works. I did not receive a response.

(7) A detailed mathematical refutation of [Boyd, 2025] can also be found in the FAQ [Joshi, 2025a].

(8) About two months ago, I once again invited Mochizuki for Zoom conversations with the hope of reconciling our respective approaches. He did not respond.

(9) This letter was written before I saw Mochizuki's response to [Boyd, 2025], but its release was delayed because [Joshi, 2025a] is currently on hold by the arxiv.org moderation team.

Sincerely,
Kirti Joshi (November 1, 2025)

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