

GROUPS AND REPRESENTATIONS II: PROBLEM SET 4
Due Tuesday, March 28

Note: Some of the problems in this problem set are from Kirillov, An Introduction to Lie Groups and Lie Algebras.

Problem 1: Kirillov Problem 8.6

Problem 2: Kirillov Problem 8.7

Problem 3: Kirillov Problem 8.9

Problem 4: Use the Weyl character formula to prove the Kostant multiplicity formula, which says that

$$\dim V_\lambda(\mu) = \sum_{w \in W} P(w(\lambda + \rho) - (\mu + \rho))$$

where $P(\mu)$ is the number of way to write μ as a sum of positive roots.

Problem 5: Use the Weyl character formula to prove the Weyl dimension formula

$$\dim V_\lambda = \frac{\prod_{\alpha \in R^+} \langle \alpha, \lambda + \rho \rangle}{\prod_{\alpha \in R^+} \langle \alpha, \rho \rangle}$$

Use this to find the dimension of the representation of $\mathfrak{sl}(3, \mathbf{C})$ with highest weight $k_1\omega_1 + k_2\omega_2$, where ω_i are the fundamental weights.