Week 1

Name: ______________________
Date: ______________________

Direct Proof

A direct proof is a proof that works directly from definitions to arrive at the desired statement using positive logical reasoning. A direct proof will NOT involve proving a logically equivalent statement, as this is an indirect proof and we will cover these next week.
We typically use a direct proof when presented with conditional statements, think statements of the form "if...then". These are commonly presented in logic as $p \implies q$ (if p, then q). Your job is to use your reasoning skills to find the $\implies$, or 'link' between your assumption, p, and your conclusion, q. This is typically done as follows:

1. Assume p to be true. Examine what you know from this assumption, ie, what are the facts you are given and what tools do you know that apply to these facts.
2. Apply tools to progressively reason towards your conclusion. Think about a maze, this step is essentially you figuring out which paths to take that lead to the end.
3. Arrive at q as a necessary result of your reasoning.

Exercises

Do your best!

1. The square of an odd number is odd.
2. The product of an even and an odd is even.
3. Is $\emptyset \in \emptyset$? What about $\emptyset \in \{\emptyset\}$?
4. How many elements are in the set $S = \{\{\emptyset, \{\emptyset\}\}, \{\emptyset\}, \emptyset\}$?
5. What is $\mathcal{P}(S)$ for the above S? Write it out fully.
6. Prove that $\emptyset \in \mathcal{P}(S)$ for any given set S.
7. Prove that for $x \in \emptyset$, then $x^2 = 9$
8. Prove that $A \cap \emptyset = \emptyset$
9. Prove that $A \cup \emptyset = A$
10. Prove that $P \vee \neg P$ is T is true regardless of the truth value of P.
11. Show that $(P \wedge Q) \vee (\neg P \vee \neg Q)$ is true.