

Mathematics V1207x
Honors Mathematics A

Practice Midterm Exam

October 28, 2015

READ AND FOLLOW CAREFULLY ALL INSTRUCTIONS BELOW

Turn off all electronic devices.

Brief visits to the men's or women's room (just out the door to your left) are OK, but one at a time only.

Write your name, "Honors Math A, Prof. Thaddeus," and the number of blue books on the cover of each blue book.

Write your name on the attendance sheet when it comes around.

Write all answers and work in your blue books. Do not hand in this sheet.

On each page you use, write the number of the problem *in a circle in the margin*.

You may do more than one problem on a single page. Just put all numbers by their problems.

You may do the problems out of order, but this is discouraged as it can lead to misgrading.

When there is any doubt, state briefly but clearly what statements from Apostol, lecture, or assignments you are using.

You may use, without comment, facts from logic and high-school algebra, and basic facts about the natural and real numbers (such as those stated without proof in Apostol).

In grading the exams, I will emphasize accuracy, brevity, and clarity. Aim for all three.

Attempt all 7 problems. Each is worth 10 points. Good luck!

1. State the approximation property of the supremum.
2. Prove that if $f : S \rightarrow T$ and $g : T \rightarrow U$ are injective, then so is $g \circ f : S \rightarrow U$.
3. Prove that for all $n \in \mathbb{N}$, $2^{n+1} \geq 2n + 2$.
4. For any $S \subset \mathbb{R}$ containing 0, let $f : [0, \infty) \rightarrow \mathbb{R}$ be $f(x) = \sup(S \cap [0, x])$.
 - (a) State a formula for f if $S = \mathbb{Z}$, the integers (no proof necessary).
 - (b) State a formula for f if $S = \mathbb{R}$, all real numbers (no proof necessary).
 - (c) Prove that any such f is integrable on $[0, 10]$.
5. For $a < b \in \mathbb{R}$, if $g : [a, b] \rightarrow \mathbb{R}$ is integrable and $|g| \leq 1$, prove that $|\int_a^b g(x) dx| \leq b - a$.
6. Prove that $\lim_{x \rightarrow 0} \frac{1}{x}$ does not exist.
7. Suppose $f, g : \mathbb{R} \rightarrow \mathbb{R}$ are any two functions such that for all $x \in \mathbb{R}$, $|f(x)| \leq |x|$ and $|g(x)| \leq 1$. Prove that the product function fg is continuous at 0.