Algebraic curves with automorphisms

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Talk, April 2017, a survey of results discussed.

Abstract. Using geometric methods on the moduli space (in characteristic zero) we try to understand which automorphism groups can appear.

The moduli space of curves can be stratified by locally closed sets on which all fibers have the same automorphism group. (We will give a precise definition.) A curve defining a stratum of dimension zero is said to have many automorphisms.

Theorem. There are infinitely many curves with many automorphisms that are not CM curves.

We try to understand minimal and maximal strata.

(Maximal strata were studied by Cornalba, we will review his results.)

Definition (a new notion). A final curve defines a minimal Hurwitz stratum. Equivalently: in a family of final curves (closed in the moduli space) there is no special fiber with strictly more automorphisms.

Clearly: "many automorphisms" implies "final". We will show there are many final curves that do not have many automorphisms (i.e. defining a positive dimensional closed stratum).

Theorem. For every curve D there exists a Galois cover $C \to D$ such that C is a final curve. (In these cases D can have a large automorphisms group, while the automorphism group of C is small.)

We will give motivation and examples. We will see that there are (infinitely many) Hurwitz strata that are minimal and maximal (for $g \to \infty$, and their dimensions are unbounded). We will give complete proofs of the results mentioned above.

Methods: *signature, monodromy, descending and lifting automorphisms in a Galois cover.*