The purpose of my talk will be to give an example of how one can use ideas from geometric Langlands correspondence for applications to "classical" algebraic geometry and mathematical physics.

More specifically let G be a semi-simple complex algebraic group. Let $\mathrm{Bun}_G(\mathbb{C}^2)$ denote the moduli space of principal G-bundles on $\mathbb{P}^2$ trivialized at ∞. We shall explain the idea of a proof of a conjecture of N.Nekrasov which relates the equivariant intersection theory on $\mathrm{Bun}_G(\mathbb{C}^2)$ (more precisely, on their Uhlenbeck compactifications) to solutions of some well-known integrable systems (this conjecture can be thought of as a mathematical formulation of the results of Seiberg and Witten from 1994). As a byproduct we reprove the results of Givental-Kim and Givental-Lee about the quantum cohomology and quantum K-theory of the flag manifolds as well as generalize those results to the case of affine flag manifolds. We shall also formulate some further conjectures which could generalize our results.

If time permits I shall briefly explain how the main ingredient of the above proof can be understood in the framework of the geometric Langlands conjecture following a suggestion of V. Drinfeld.

The talk is based on a series of joint works with various collaborators (including P. Etingof, M. Finkelberg and D. Gaitsgory). No previous knowledge of either subject will be assumed.