The lubrication approximation leads to a fourth order degenerate equation modeling the evolution of small viscous droplets on a solid support (the thin film equation). Along the contact line (aka the free boundary), the solution must satisfy a gradient condition (contact angle condition). While many existence and regularity results are known for solutions with zero contact angle, the only existence result with non-zero contact angle is due to Otto and only holds in some particular framework (Hele-Shaw cell). Following Bertsch, Giacomelli and Karali, we consider a regularization of this free boundary problem to attempt to generalize Otto's result.