The Monge-Ampere equation arises in connections with several problems from geometry and analysis (optimal transport, the Minkowski problem, the affine sphere problem, etc.) The regularity theory for this equation has been widely studied. In particular, in the 90's Caffarelli developed a regularity theory for Alexandrov/viscosity solutions, showing that strictly convex solutions of $det(D^2u) = f$ are $C^{1,\alpha}$ provided f is bounded away from zero and infinity, and are $W^{2,p}$ with $p > 1$ if f is uniformly close to a constant. The counterexamples by Wang in 1995 showed that the results of Caffarelli were more or less optimal. However, an important question which remained open was whether solutions with right hand side bounded away from zero and infinity could be $W^{2,1}$. In a recent joint work with De Philippis we prove that this is the case: indeed, not only solutions are $W^{2,1}$, but that actually $|D^2u|$ belongs to $L\log^k L$ for any $k > 0$. In this talk I'll review Caffarelli's $C^{1,\alpha}$ regularity theory and show how to prove $W^{2,1}$ regularity.