Background

$$\begin{array}{ccc} \Delta' & & \Delta \\ N' & \xrightarrow{f} & N \end{array}$$

if $\forall$ cone $\sigma' \in \Delta'$ $\exists$ $\sigma \in \Delta$ s.t. $f(\sigma') \subseteq \sigma$

we get a map $X_{\Delta'} \xrightarrow{\varphi} X_{\Delta}$

Def: φ is proper if $f^{-1}(|\Delta|) = |\Delta'|$

Want: $N \xrightarrow{id} N$

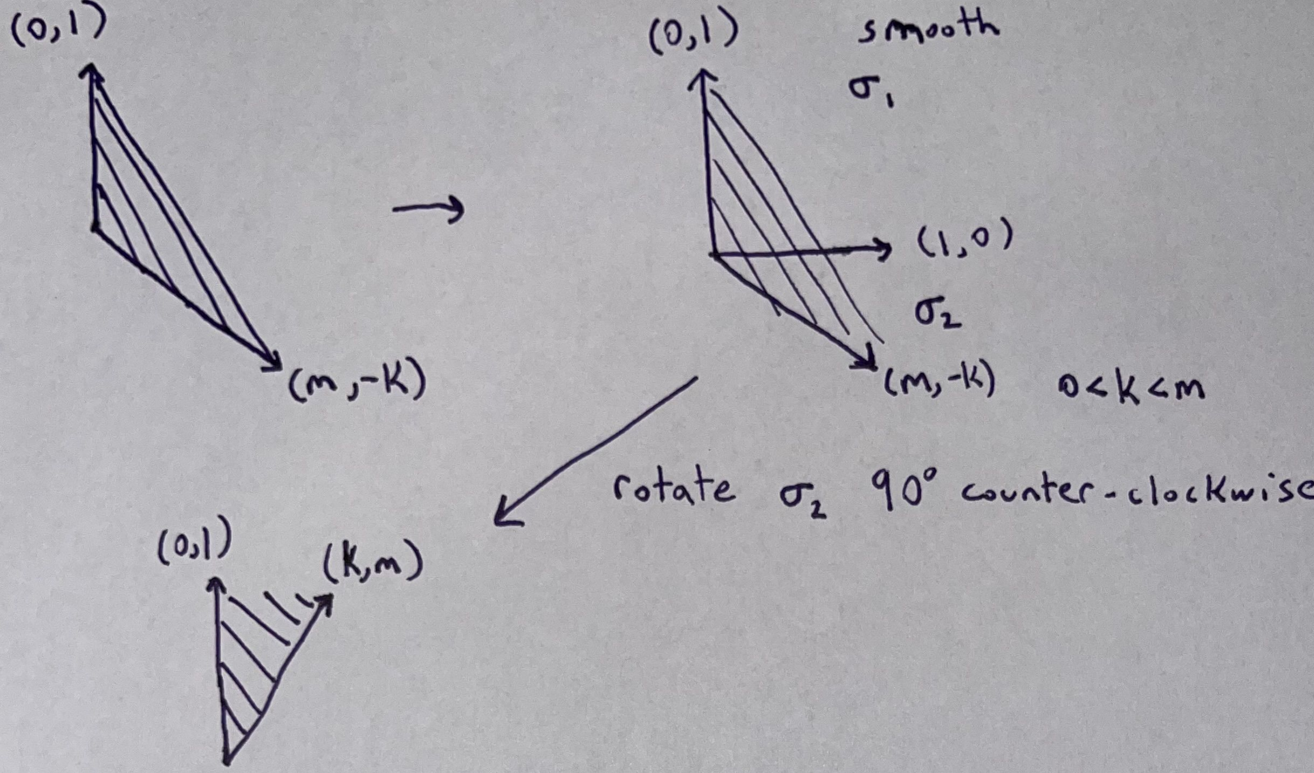
$$\begin{array}{ccc} \Delta' & & \Delta \end{array}$$

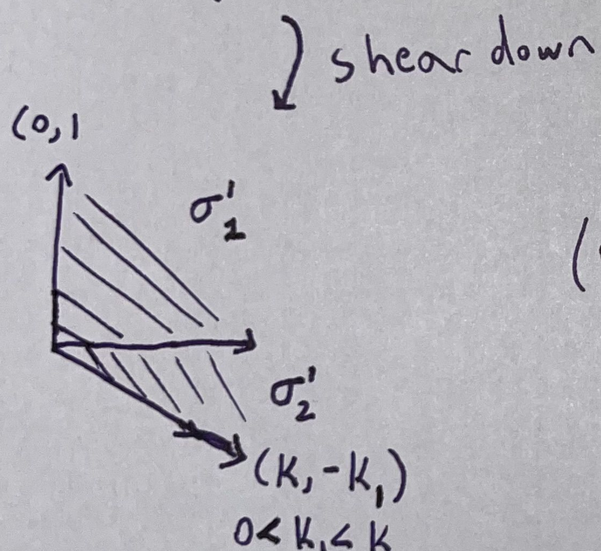
s.t. $\forall \sigma' \in \Delta'$ $\exists$ $\sigma \in \Delta$ s.t. $\sigma' \subseteq \sigma$

(subdivision of Δ)

s.t. every cone in Δ' is nonsingular

(generated by part of a basis for N)





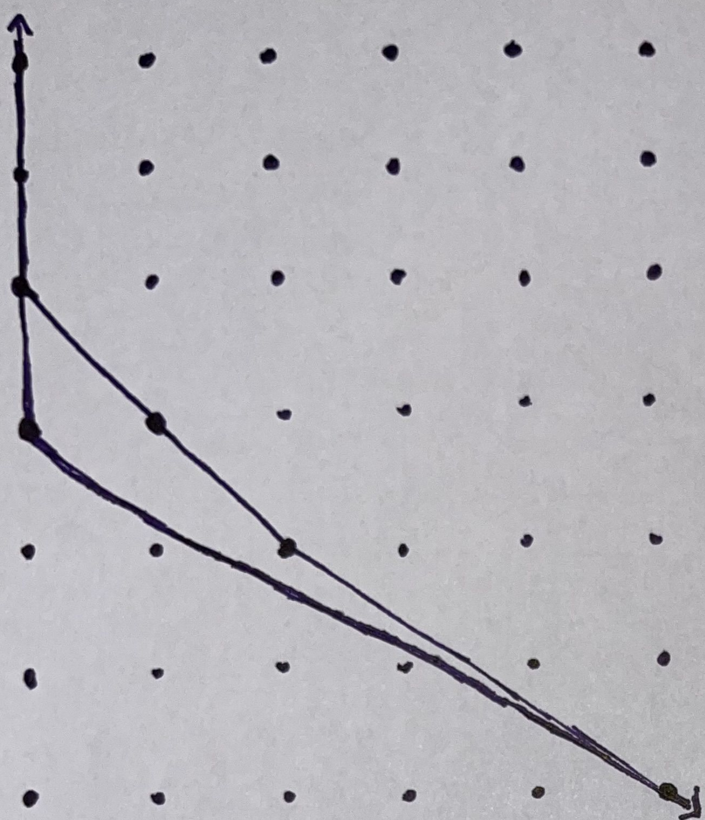
(repeatedly apply $\begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}$)

$$\begin{pmatrix}1 & 0\\ -a & 1\end{pmatrix}\begin{pmatrix}k\\ m\end{pmatrix} = \begin{pmatrix}k\\ m-ak\end{pmatrix}$$

$$-k_1 = m - a_1 \cdot k$$

$$\Rightarrow k_1 = a_1 \cdot k - m$$

$$\Rightarrow \frac{k_1}{k} = a_1 - \frac{m}{k} \Rightarrow \frac{m}{k} = a_1 - \frac{k_1}{k} = a_1 - \frac{1}{\frac{k}{k_1}}$$



ex:

$m = 5$

$K = 3$

$a_1 = 2$

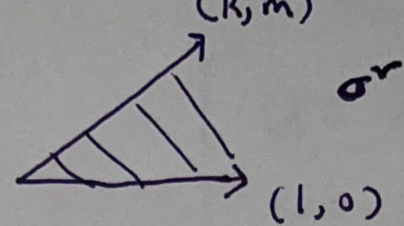
$K_1 = 1$

$1 = 2 \cdot 3 - 5$

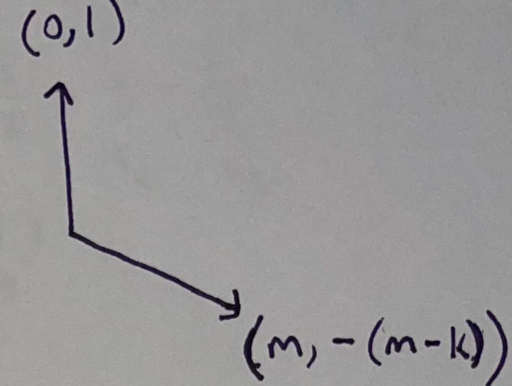
Intuition: Hirzebruch-Jung continued fraction gives a better and better approximation of $\frac{m}{K}$ of ray through $(m, -K)$

Strategy

Start)



1) Apply $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \rightsquigarrow$



2) Apply the same process as before to get $\frac{m}{m-k} = b_2 - \cfrac{1}{b_3 - \cfrac{\ddots}{\cdots \cfrac{1}{b_n -}}}$

Convex hull of non zero lattice points

3) Apply $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

powers of x ↓ powers of y ↓

to get recursion for k_i, l_i

$k_1 = 1 \quad k_2 = 1 \quad k_{i+1} = b_i \cdot k_i - k_{i-1}$

$l_1 = 0 \quad l_2 = 1 \quad l_{i+1} = b_i \cdot l_i - l_{i-1}$

$\mathbb{C}[S_\sigma] = \mathbb{C}[x^{k_i} y^{l_i}]$

4) Set $u = x^{1/m}$

$v = x^{k/m} y$

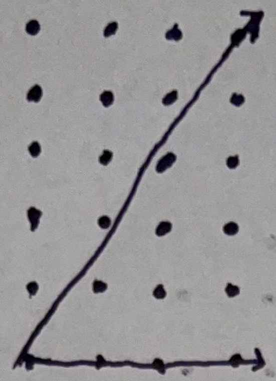
↓

$s_1 = m \quad s_2 = m - k \quad s_{i+1} = b_i \cdot s_i - s_{i-1}$

$t_1 = 0 \quad t_2 = 1 \quad t_{i+1} = b_i \cdot t_i - t_{i-1}$

generators

$\mathbb{C}[S_\sigma]$



$$\begin{bmatrix} u = x^{1/5} \\ v = x^{3/5}\, y \end{bmatrix}$$

$$\frac{5}{2} = \underset{b_2}{3} - \underset{b_3}{\frac{1}{2}}$$

$(1,0)$	$(1,1)$	$(2,3)$	$(3,5)$
x	xy	x^2y^3	x^3y^5
u^5	$u^2 \cdot v$	$u \cdot v^3$	v^5
$\binom{m}{0}$	$\binom{2}{1}$	$\binom{1}{3}$	$\binom{0}{5}$