Decoherence and Reecoherence

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1 Introduction

This seminar talk will cover the following:

- Discussing the primary obstacle in quantum systems realization: decoherence. Additionally, examining fundamental strategies for addressing this challenge through introductory error correction approaches such as the Shor quantum code, an expansion of the traditional three-bit repetition code.
- Highlighting the discrepancy between theory and practical implementation in quantum computing. Theoretical knowledge suggests starting with elementary quantum logic gates and integrating them into more complex quantum networks.
- Addressing the practical difficulties encountered when more quantum gates are networked together, leading to significant operational challenges due to increased qubit interactions. This makes it increasingly difficult to isolate the qubits from environmental entanglement.
- Decoherence is identified as a critical issue that compromises the unique interference abilities of quantum computers, thereby reducing their computational power.
- Introducing error correcting codes as a solution to combat decoherence. These codes are designed to protect data from errors by distributing the information across a greater number of ancillary qubits.

2 The Three-Qubit Code

Isometries and Quantum Error Correction:

- Isometric operators V map one Hilbert space to another and satisfy $VV^\dagger = I$. These isometries can be reversed or corrected by applying $V^\dagger$.
- A quantum channel $\mathcal{E} : B(\mathcal{H}) \to B(\mathcal{H}')$ is correctable if there exists a recovery channel $\mathcal{R} : B(\mathcal{H}') \to B(\mathcal{H})$ such that $\mathcal{R} \circ \mathcal{E}$ is the identity on the input space of the channel.

Correctable Channels and Isometries:

- Only completely correctable channels are those equivalent to statistical mixtures of isometries that are mutually orthogonal.
- Visualization of correctable (left) and non-correctable (right) channels, where each isometry V_i leads to different spaces with some probability p_i.

The Three-Qubit Code Example:

- Encoding a qubit state $\alpha|0\rangle + \beta|1\rangle$ into three qubits by introducing two ancilla qubits in state $|00\rangle$ and applying a unitary operation that entangles them, resulting in the state $\alpha|000\rangle + \beta|111\rangle$.
- This encoding is an isometric embedding of the original qubit's Hilbert space into the Hilbert space of the three qubits, and can be reversed with the mirror image circuit.

Decoding and Error Detection:

- Given four isometries V_i which form the output of the channel, one can reverse the encoding to retrieve the original state if the particular isometry used is known.
- Projections on specific subspaces defined by the isometries allow for error detection and correction by applying corresponding projections in the respective subspace.

Visualizing Isometries and Correctable Channels:

- The illustration shows how four isometries can be implemented and how to reverse these operations in the error correction process.

3 Towards Error Correction

Inverting Quantum Channels:

- In quantum error correction, the act of choosing a random isometry from a set can be considered as the encoding step, resulting in a specific encoded state.

Expressing Isometries with Unitary Operators:

- Expressing four isometries V_i for encoding a single qubit into a three-qubit state can be written using tensor products of unitary operators and V_{00}.
- The encoding process is then perceived as randomly picking between four distinct noise processes, each represented by an isometry followed by a noise operation.

Error Correction Process:

- Error correction involves identifying which specific error has occurred, fixing it, and then reversing the encoding. This process is known as decoding.
- Quantum error correction can be visualized as a three-step process: encoding, noise introduction through a channel, and error detection followed by decoding.

Stabilizer Formalism:

- The stabilizer formalism provides a natural way to describe error-correcting codes, using stabilizers to define the code space.
- The correctable space (without error) and the error can be identified by measuring the stabilizer values. Identifying the error involves determining which of the possible errors (related to the stabilizers) has occurred based on the measurement outcomes.

Generalizing Isometries for Error Correction:

- If a set of correctable isometries are related by $V_i = U_i V_0$ for some set of unitaries $\{U_i\}$ with $U_i U_j = \delta_{ij}$, then the encoding operation V_0 provides protection against the errors U_i.

4 Understanding Quantum Errors as Discrete Events

- The entanglement of a quantum computer with its environment results in a discernible trail of the computation within the environment, which is pivotal in understanding quantum errors.
- Quantum computation can lead to distinct environmental outcomes, represented by two states $|0\rangle_{env}|0\rangle_c$ and $|1\rangle_{env}|0\rangle_c$, indicating which computational path was taken.
- In the absence of decoherence, two final states are indistinguishable unless the environment retains no information about the computational path.
- Decoherence is understood by tracing over the environment, evolving the state $|\psi(t)\rangle$ to a mixed state, which is achieved by observing the environment's influence on the quantum state.
- As the environmental interaction increases (as captured by a parameter ϵ), the system evolves from a pure state to a mixed state, signifying the occurrence of decoherence.
- This process is conceptualized as quantum computation leaking into the environment, transforming the computational process from a coherent superposition into discrete outcomes.

Mathematical Representation:

- The evolution of a quantum state influenced by the environment can be expressed as:

$$|\psi(t)\rangle \to \alpha|0\rangle_c|f_0(\epsilon)\rangle_{env} + \beta|1\rangle_c|f_1(\epsilon)\rangle_{env}, \tag{1}$$

  where $|f_i(\epsilon)\rangle_{env}$ are the environmental states correlated with the computational basis states $|i\rangle_c$.
- When $\epsilon \to 0$, the system tends toward no environmental interaction, preserving quantum coherence.
- The limit of this process as ϵ approaches zero relates to the idea of discretizing quantum errors and how we perceive them in computational operations.

Stabilizer Formalism:

- The stabilizer formalism provides insight into error correction, where errors are identified by their impact on the stabilizers of the quantum state.
- In practice, we measure stabilizers to detect and correct errors, ensuring the quantum state remains in the correct computational subspace.

Concluding Thoughts:

- Decoherence showcases how quantum computation shifts from an ideal, coherent state to an error-prone, discretized state due to environmental interactions.
- Understanding these discrete errors is essential for developing effective quantum error correction techniques.

5 Digitizing Quantum Errors

- The interaction between a qubit and its environment can be represented by a general transformation, which impacts the qubit's state by entangling it with the environment's states.
- When environmental states are neither normalized nor orthogonal, decoherence arises, leading to a mixed state of the quantum system.

Representing Environmental Impact:

- The effect of the environment on a qubit in a superposition state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ can be modeled as:

$$|\psi\rangle \rightarrow \frac{|\psi\rangle|e_0\rangle + Z|\psi\rangle|e_1\rangle + X|\psi\rangle|e_2\rangle + Y|\psi\rangle|e_3\rangle}{2}, \tag{2}$$

where X, Y, Z are the Pauli matrices representing bit-flip, phase-flip, and both errors respectively, and $|e_i\rangle$ are the environmental states.

Types of Quantum Errors:

- Four primary events can occur to the qubit due to environmental interaction: no change, phase-flip, bit-flip, and both bit and phase flip.
- Accurate error identification and correction depend on the distinguishability of the environmental states.

Simplifying Error Correction:

- Quantum errors can be reduced to two fundamental types: bit-flip and phase-flip errors.
- Pauli error correction is pivotal as it implies that correcting Pauli errors is sufficient to correct all possible errors.

Quantum Error Evolution:

- For n qubits in a state $|\psi\rangle$, and an environmental state $|e\rangle$, the evolution can be expressed as a sum over tensor products of Pauli operators and environmental states:

$$|\psi\rangle \rightarrow \sum_i E_i|\psi\rangle|e_i\rangle, \tag{3}$$

where E_i are tensor products of the Pauli operators acting on n-qubits.

Kraus Operators and Quantum Channels:

- Quantum channels can be described using Kraus operators E_i, transforming a state ρ into a mixed state through their action.

- The probability of each error occurring is related to the square of the norm of the corresponding environmental state in the mixture.

Closing Remarks:

- The discretization of quantum errors is crucial for quantum error correction and can be described using the formalism of quantum channels and Kraus operators.
- Understanding these concepts allows us to devise strategies to mitigate the effects of decoherence and maintain the fidelity of quantum computations.

6 Recoherence in Quantum Systems

- Recoherence aims to undo the entanglement caused by decoherence between a quantum system and its environment.
- If we could observe the environment without disturbing the system, we could revert to the original state before decoherence. In practice, we work with a subsystem that we control, called an ancilla.

Coupling to an Ancilla:

- We manage decoherence by coupling our quantum system to an ancilla that we control, which then interacts with the environment. This process prepares the ancilla in a specific state, attempting to reverse the decoherence.

Decoherence and Recoherence Process:

- The combined action of decoherence and subsequent recoherence can be expressed as:

$$R(\mathcal{E}(|\psi\rangle\langle\psi|)) = \sum_{i,j} R_{ij} E_j |\psi\rangle\langle\psi| E_i^\dagger R_{ij}^\dagger, \tag{4}$$

  where E_i are error operators due to decoherence, and R_{ij} are recoherence operators.

Ideal Outcome:

- The desired outcome is to disentangle the qubits from the environment and have the ancilla entangled with nothing else, achieving a state like $|a\rangle$ (a pure ancilla state).

Practical Challenges:

- Achieving this ideal disentanglement for all states is too challenging. We focus on a subset of recoverable states belonging to the codespace.

Error Correction via Recoherence:

- The recoherence operator is designed to correct certain errors. If it can correct Pauli errors, it can also correct a combination of such errors.

Correcting Quantum Errors:

- A successful quantum error correction method that corrects errors E_i and E_j will also correct their combination.

- For example, a three-qubit encoding corrects single-bit errors. If a system can correct these, it can correct a wider range of errors, demonstrated by the relationship:

$$R_{a,b} = \delta_{a,b} A_1 \tag{5}$$

where A_1 is a recovery operator that acts on the codespace.

Closing Notes:

- Recoherence is the counterpart to decoherence, providing a pathway to recover information and maintain the integrity of a quantum state.
- Understanding and implementing recoherence is a pivotal step towards reliable quantum computation and error correction.