

PROBLEM SET 2

$$G = \mathrm{PGL}_2.$$

Problem 1. *Verify Drinfeld–Gaitsgory 2nd adjointness when evaluated on the constant sheaf k_{Bun_G} on Bun_G .*

Problem 2. *Let B^- be the standard opposite Borel subgroup and define $\mathrm{CT}_*^-, \mathrm{CT}_!^-$ as in the lecture but replacing B with B^- . Show that Drinfeld–Gaitsgory 2nd adjointness can be rewritten as*

$$\mathrm{CT}_!^- \simeq \mathrm{CT}_*^-.$$

Now try to make a guess for the natural transformation $\mathrm{CT}_ \rightarrow \mathrm{CT}_!^-$. Challenge: make a guess for the other direction $\mathrm{CT}_!^- \rightarrow \mathrm{CT}_*$.*

Problem 3. *Challenge: for $g \geq 1$, show that cuspidal sheaf exists.*

Problem 4. *Find a nonzero \mathcal{F} such that $\mathrm{coeff}_0(\mathcal{F}) \simeq 0$.*