Problem :

1. Consider $X = PGL_2 \times PGL_2 / PGL_2$.
Show that $\bar{X} \simeq \mathbb{P}^3$. Describe the embedding
$N \times \bar{A} \hookrightarrow \mathbb{P}^3$.

2. Assume $K = G^\theta$ is connected. Show that $\langle 2\rho, \lambda \rangle$ is even for $\lambda \in X_\bullet(A)^+$. Give a counter-example when K is disconnected.

3. Consider $X = GL_{2n} / Sp_{2n}$.

① Construct a placid presentation of LX.

② Describe $S^\circ_{\leq \lambda}$ where $\lambda = \underbrace{(n-1, n-1, -1, -1, \cdots, -1)}_{2n} \in X_\bullet(A)^+$.

4. Let $G = GL_n$, $K = O_n = G^\theta$, $X = GL_n / O_n$,
$\theta(g) = ({}^t g)^{-1}$.

① Show that there are two LG-orbits in LX, and classify which L^+G-orbits are in the same LG-orbit.

② Show that stabilizers of L^+G-orbits are all disconnected
($\Rightarrow \exists$ non-constant local system on each orbit).