Lecture I.

Problem 1:

Show $SO_{2n+1}^{\vee} = Sp_{2n}$

$SO_{2n}^{\vee} = SO_{2n}$.

Problem 2:

Show $\dim Gr^{\lambda} = \langle 2\rho, \lambda \rangle$.

2ρ = sum of positive roots.

Problem 3: (Lusztig Embedding)

$G = GL_n$, $2\rho = (n-1, n-3, \cdots, -n+1)$

$\mathcal{N}_n$ = $n \times n$ nilpotent matrices $\quad X_*(T)^+ = \{(\lambda_1, \cdots, \lambda_n) \in \mathbb{Z}^n \mid \lambda_1 \geq \cdots \geq \lambda_n\}$.

1. $\dim \mathcal{N}_n = \langle 2\rho, (n, 0, \cdots, 0) \rangle = n(n-1)$.
2. $\phi : \mathcal{N}_n \xhookrightarrow{\text{open}} \overline{Gr^{(n,0,\cdots,0)}}$
   $X \longmapsto (t-X)L_0$ is an open embedding.
3. $\phi(\mathcal{O}_\lambda) \subset Gr^{\lambda}$ $\quad \lambda = (\lambda_1, \cdots, \lambda_n)$, $\sum \lambda_i = n$, $\lambda_i \geq 0$.
   $\mathcal{O}_\lambda$: nilpotent orbit of type λ.

Problem 4: $G = GL_2$.

1.
$$\begin{tikzcd}
\tilde{\mathcal{N}}_2 \arrow[r, hook] \arrow[d, "\text{Springer resolution}"'] & Gr^{(1,0)} \tilde{\times} Gr^{(1,0)} = LG^{(1,0)} \overset{L^+G}{\times} Gr^{(1,0)} \arrow[d] \\
\mathcal{N}_2 \arrow[r, hook, "\text{open}"] & \overline{Gr^{(2,0)}}
\end{tikzcd}$$

2. $IC_{(1,0)} * IC_{(1,0)} \simeq IC_{(2,0)} \oplus IC_{(1,1)}$.

③ Calculate weight spaces of

$$H^*(Gr_G, IC_{(1,0)} * IC_{(1,0)}) \cong H^*(Gr_G, IC_{(2,0)}) \oplus H^*(Gr_G, IC_{(1,1)}).$$

and compare with

$$L(1,0) \otimes L(1,0) \cong L(2,0) \oplus L(1,1)$$