Contents

I Deterministic Control **5**

1 Static Optimization **7**
1.1 Calculus Reminder 7
1.2 Abstract Results 8
1.3 Euler Conditions 9
1.4 Kuhn–Tucker Theorem 11

2 Calculus of Variations **15**
2.1 Problem without Terminal Condition 17
2.2 Sufficient Condition for Optimality 18
2.3 Example: 1-Dimensional Quadratic Problem 19
2.4 Example: Optimal Consumption 19

3 Maximum Principle **21**
3.1 Bolza, Lagrange and Mayer formulation 22
3.2 Pontryagin Maximum Principle 23
3.3 Sufficient Condition for Optimality 25
3.4 Example: Linear Quadratic Regulator 27
3.5 Example: Consumption and Savings 28

4 Dynamic Programming **31**
4.1 Introductory Example 32
4.2 Dynamic Programming Principle (DPP) 33
4.3 Dynamic Programming Equation (DPE) 35
4.4 Verification 38
4.5 DPE and Maximum Principle 39
4.6 Example: Linear Quadratic Regulator 40
4.7 Example: Optimal Consumption 41

II Stochastic Control **43**

5 Stochastic Analysis Reminder **45**
5.1 Stochastic Differential Equations 45
5.2 Feynman–Kac Representation 47

6 Dynamic Programming in Standard Form **49**
6.1 Dynamic Programming Equation 54
6.2 Verification 57
6.3 Infinite Time Horizon 58

7 Portfolio Choice **63**
7.1 Black–Scholes 64
7.2 Example with Stochastic Volatility 66
7.3 Logarithmic Utility 69

8 Optimal Stopping **73**
8.1 Essential Supremum 73
8.2 Snell Envelope Theory in Discrete Time 74
8.3 Snell Envelope Theory in Continuous Time 79
8.4 Optimal Stopping of a Diffusion 80
8.5 Examples 88