Reminder of knot theory so far:

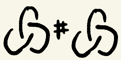
- Knots, links and their invariants: tricolorability, Jones polynomial, crossing number...

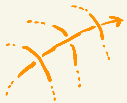
- Computational knot theory: ,

- Special kinds of knots (or knot diagrams):

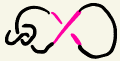
prime, alternating, reduced, connected, amphichiral

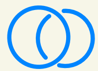
NOT

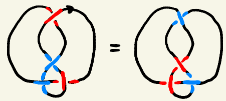




NOT





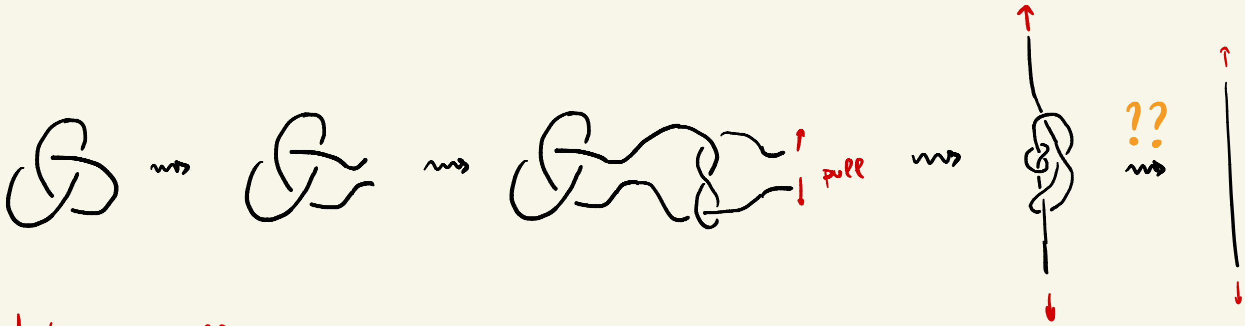


- Tait's conjectures

So far: mostly distinguishing knots

Today: a new question and a new dimension

New Question: Can you untie a knot by knotting it more?



Intuition (poll):

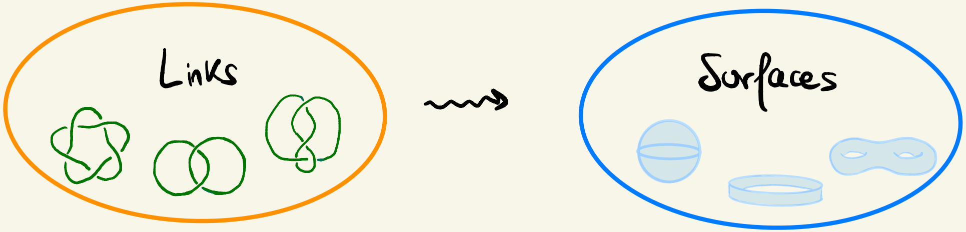
Mathematical formulation: are there two (proper) knots K_1, K_2 such that

$$K_1 \# K_2 = \text{unknot?}$$

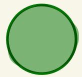
Spoiler: No. But how do we prove it?

8. One dimension up: Surfaces

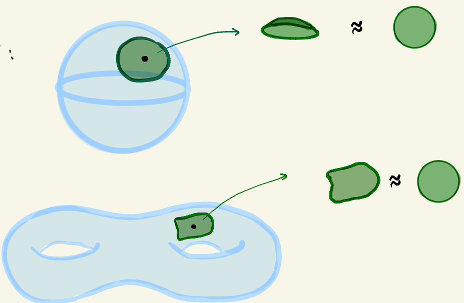
We will associate



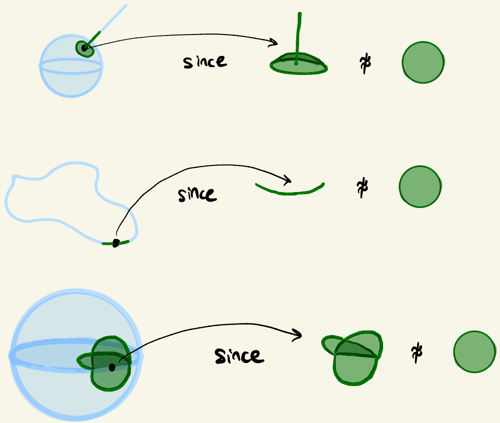
Definition: a surface without boundary is a subset of 3D space such that around every point there is a "disk": a copy of $B^2 = \{x^2+y^2 \le 1\}$ =



Examples:



Nonexamples:

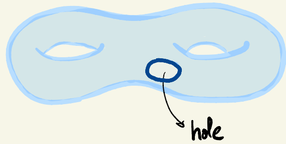


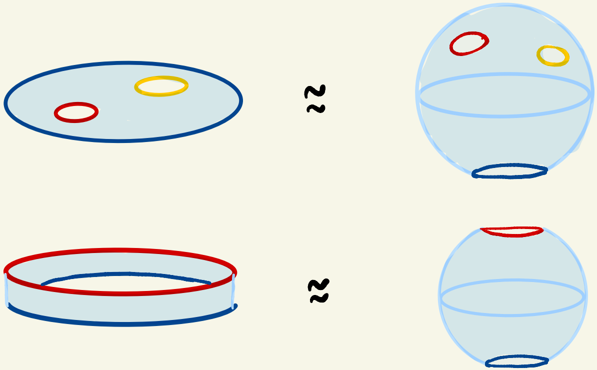
"Local condition"

Q?

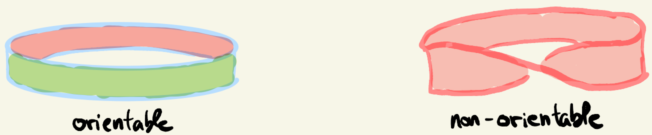
Definition: a surface with boundary is the space obtained by removing one or more disks from a surface without boundary

Examples:





Warning: we will not consider non-orientable surfaces, that is, surfaces that cannot be painted with two colors:

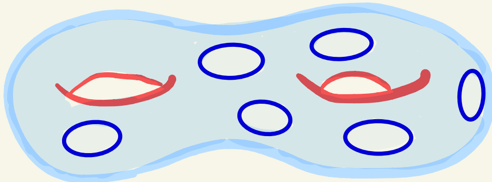


Q?

Fact: a surface is determined by

number of "donut holes" genus g | number of removed discs #boundary components b

Example:

$T_{g,b} =$ 

is the surface with genus $g = 2$

and #bdary components $b = 6$

Technical remark: just like with knots, we may stretch surfaces without tearing them apart – this does not change the surface.

Q?