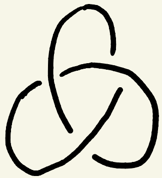
3. Multiplying knots, table of prime knots

 #  = ?

Analogy: Integers

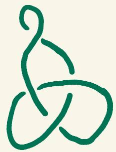
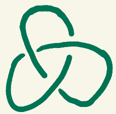
Primes

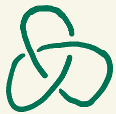
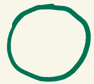
Knots

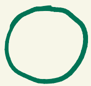
Prime knots

Theorem: Every knot decomposes uniquely as a product of prime knots.

<u>Definition</u> (Crossing number): The crossing number of a link is the minimum amount of crossings that a link diagram can have.

Example: cr () $\overset{RI}{=}$ cr ()

Now if this is <3, then  =  , since we saw a knot with ≤ 2 crossings is the unknot

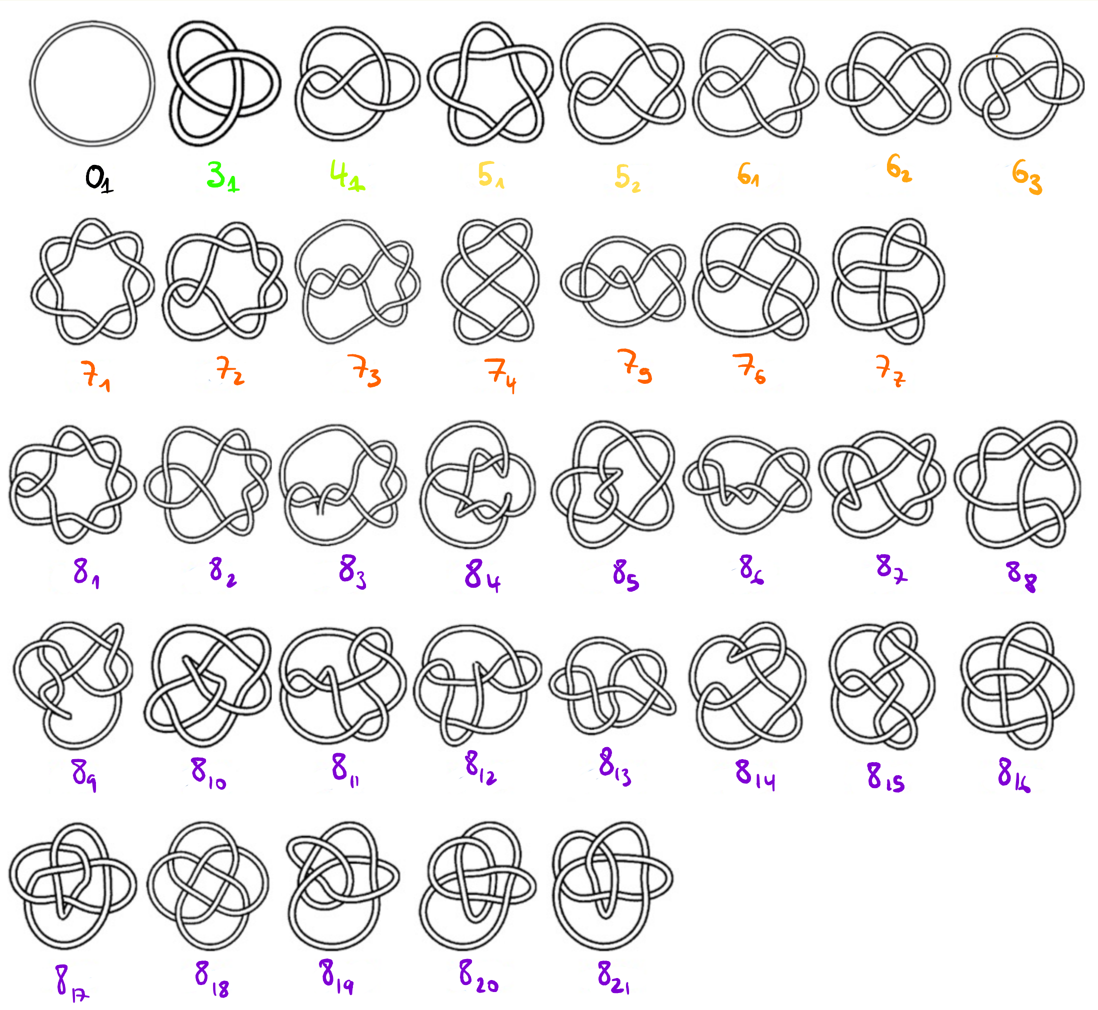
However  $\neq$  $\Rightarrow$ cr () $= 3$. $\blacksquare$

tricolorable   not tricolorable

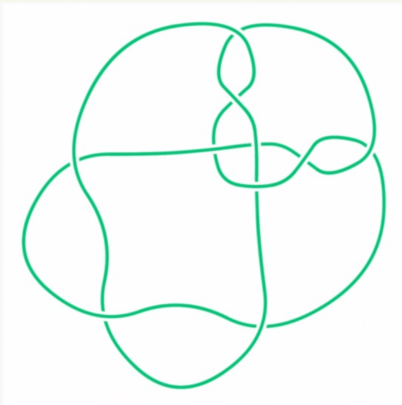
Q?

Idea: we can classify knots with crossing number n.

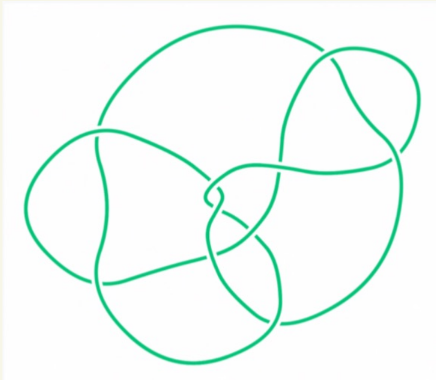
TABLE OF PRIME KNOTS UP TO 8 CROSSINGS



- All prime knots have been classified up to 16 crossings
- Number of knots increases rapidly: http://oeis.org/A002863
- Cautionary tale: the Perko pair



10_{161}



10_{162}

Some open questions:

- Do we have that $cr(K_1 \# K_2) = cr(K_1) + cr(K_2)$?
- Is there an efficient algorithm to identify a random diagram with one from the table?

Let $f(n)$ = # prime knots with crossing number n.

- What is $f(17)$?
- Is there a formula for $f(n)$?
- Is it true that $f(n+1) > f(n)$ always?

Q? Exercises (rest)