MATH V1201 SECTIONS 002 & 003 HOMEWORK 6
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1. Some Stewart problems

(I.1) Stewart 2.4.11.
(I.2) Stewart 2.4.12.
(I.3) Stewart 2.3.35. See Problem (V.2) for the graphing part.

2. A brief aside

Why is $\lim_{x\to 0}\frac{x^2}{x}=0$? Because: for $x\neq 0$, $\frac{x^2}{x}=x$, and $\lim_{x\to 0}f(x)$ only depends on the value of $f(x)$ *for* $x\neq 0$. So, $\lim_{x\to 0}\frac{x^2}{x}=\lim_{x\to 0}x=0$. (The last equality is because $f(x)=x$ is a continuous function at 0.) With this in mind...

(II.1) What is $\lim_{(x,y)\to(0,0)}\frac{0}{x^2+y^4}$? Explain your answer.

3. Showing limits exist using the squeeze theorem

Use the squeeze theorem to prove that each of the following limits exists (and, in the process, find what the limit is equal to):

(III.1) $\lim_{x\to 0}\frac{x^2}{x+x^2}$.
(III.2) $\lim_{x\to 0}\frac{\sin(x)^3}{x^2}$.
(III.3) $\lim_{x\to 0}\frac{\sin(x^3)}{x^2}$.
(III.4) $\lim_{x\to 0}\frac{x^3+\sin(x)^5}{x^2+x^3}$.

4. Showing limits don't exist

Show that each of the following limits does not exist, by approaching from several directions:

(IV.1) $\lim_{(x,y)\to(0,0)}\frac{x^2y}{x^3+3y^5}$
(IV.2) $\lim_{(x,y)\to(1,2)}\frac{xy-2}{x^2-2x+y^2-4y+5}$
(IV.3) $\lim_{(x,y,z)\to(0,0,0)}\frac{xyz^2}{x^4+y^4+z^4}$.
(IV.4) $\lim_{(x,y,z)\to(1,2,3)}\frac{\sin(\pi xy)z^4}{(x-1)^2+(y-2)^4+(z-3)^6}$

5. Mathematica

We have seen a number of examples of weird behavior of limits. Let's plot what some of those functions look like.

(V.1) To graph the function $f(x) = x^2$, for x from -1 to 2, you would use `Plot[x^2,{x,-1,2}]`. Try it.

(V.2) Do the graphing part of Stewart 2.3.35, using the Plot function. (See also Problem (I.3).)

(V.3) You plot functions of two variables with the `Plot3D` function. For example, the function $f(x, y) = x^2 + y^2$ is plotted with:

```
Plot3D[x^2+y^2,{x,-10,10},{y,-10,10}]
```

Try it. (The arguments `{x,-10,10},{y,-10,10}` give the x-range and the y-range of the plot, of course.)

(V.4) Sometimes, Mathematica chooses a lousy plot range. For example, when I run:

```
Plot3D[Sin[y]/(1+x^2),{x,-10,10},{y,-10,10}]
```

The graph looks truncated. Using instead

```
Plot3D[Sin[y]/(1+x^2),{x,-10,10},{y,-10,10}, PlotRange->{{-10,10},{-10,10},{-2,2}}]
```

fixes the problem. Try both.

(V.5) Recall that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist because approaching $(0, 0)$ along different lines gives different answers. Plot $f(x, y) = \frac{xy}{x^2+y^2}$.

(V.6) Recall that $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ does not exist because even though approaching $(0, 0)$ along any line gives 0, approaching along the parabloa $y = x^2$ doesn't. Plot $f(x, y) = \frac{x^2y}{x^4+y^2}$ and see what's going on. (Adding the directive `PlotPoints -> 100` in your `Plot3D` function will help you see the plot.)

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