MATH V1201 SECTIONS 002 & 003 HOMEWORK 4
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1. Review

(I.1) Let $f(x)$ be the sine of x degrees (rather than radians). What is $f'(x)$?

2. Some Stewart problems

(II.1) Stewart 13.1.37.
(II.2) Stewart 13.1.42.
(II.3) Stewart 13.2.53.

3. Mathematica

(III.1) You already know how to draw parametric curves, from when we were drawing lines. For example, here's a semi-circle and a helix:

```
ParametricPlot[{Sin[t], Cos[t]}, {t, 0, Pi}]
ParametricPlot3D[{Sin[Pi*t], Cos[Pi*t], t}, {t, -2, 4}]
```

(Try 'em.)

(III.2) You can also do parametric plots in polar coordinates directly. Here is the curve $r = \theta$ (in the plane):

```
PolarPlot[theta, {theta, 0, 2*Pi}]
```

(Try it.)

(III.3) To differentiate a function, use the operator D. You have to indicate the dependent variable you want to differentiate with respect to. For example, to differentiate e^{t^2} use

```
D[Exp[t^2], t]
```

(Try it.)

(III.4) Differentiating vector-valued functions works just the same: to differentiate $\langle \sin(\pi t), \cos(\pi t), t\rangle$ use

```
D[{Sin[Pi*t], Cos[Pi*t], t}, t]
```

(Try it.)

(III.5) For indefinite integrals, use `Integrate`. For example:

```
Integrate[x*Sin[x^2], x]
Integrate[{t, t^2, t^3}, t]
```

(Try them.)

(III.6) For definite integrals, there are different commands depending on whether you want an exact answer or a numerical approximation. For an exact answer, try:

```
Integrate[{t, t^2, t^3}, {t,0,2}]
Integrate[Tan[x^2], {x, 0, 1}]
```

(Try them.)

(III.7) You will notice that the second answer is not very enlightenting. To get a numerical approximation, try:

```
NIntegrate[Tan[x^2], {x, 0, 1}]
```

(Try it. The "N" is for "numerical", I guess.)

(III.8) Use Mathematica to check your answers to WebAssign Homework 8 Questions 3 and 8. (Note: the Mathematica function for natural logarithm is `Log[x]` and for exponential is `Exp[x]`.)

(III.9) Use Mathematica's `Integrate` to solve Stewart Exercises 13.3.5 ("Compute the length of $\vec{r}(t) = \langle 1, t^2, t^3\rangle$, $0 \leq t \leq 1$").

(III.10) Use Mathematica's `NIntegrate` to solve Stewart Exercise 13.3.8 ("Compute the length of $\vec{r}(t) = \langle t, e^{-t}, te^{-t}\rangle$, $1 \leq t \leq 3$").

(III.11) Use Mathematica's built-in function `ArcLength` to check your answer to Problem (III.9). (As an example, the arc length of $\vec{r}(t) = \langle t, t^2, t^3\rangle$ from 2 to 5 is `ArcLength[{t, t^2, t^3}, {t, 2, 5}]` .)

(III.12) A *smooth function* $f\colon \mathbb{R} \to \mathbb{R}$ is a function that you can differentiate any number of times, i.e., so that $f^{(n)}(x)$ exists for any $n \geq 0$ and any $x \in \mathbb{R}$. So, you might want to define a *smooth curve* $\vec{r}\colon \mathbb{R} \to \mathbb{R}^3$ as a vector-valued function so that the n^{th} derivative $\vec{r}^{(n)}(t)$ exists for any $n \geq 0$ and any $t \in \mathbb{R}$. This definition is not really satisfactory. Consider the curve

$$\vec{r}(t) = \langle t^2, t^3, 0\rangle.$$

(a) Use Mathematica to plot $\vec{r}(t)$ on for $-2 \leq t \leq 2$ and notice that it has a fairly wicked kink (i.e., doesn't look smooth).

(b) Compute the derivatives $\vec{r}^{(n)}(t)$ for all $n \geq 0$. In particular, they all exist. (To compute the third derivative of $\langle \sin(x), \cos(x)\rangle$, say, using Mathematica, use `D[{Sin[x],Cos[x]},{x,2}]` .)

(c) Compute $\vec{r}'(0)$, by hand or using Mathematica. (To evaluate the expression $\langle x^2, x^3\rangle$ at $x = 5$, say, use `{x^2,x^3}/.x->5` .)

(A better definition of a smooth curve is a curve so that $\vec{r}'(t)$ exists and is never the zero-vector $\vec{0}$.)

(III.13) Optional: write a Mathematica function which takes as input a vector-valued function (of t), a t-range, and a value of t and draws both the parametric curve and the tangent line at the given t value. So, for example, I should be able to run:

```
PlotWithTangentLine[{Sin[Pi*t], Cos[Pi*t], t}, {t, -2, 4},1.7]
```

to get a plot of the helix together with its tangent line at $t = 1.7$.

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