MATH V1201 SECTIONS 002 & 003 HOMEWORK 2
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ROBERT LIPSHITZ

1. Making sure you have notation straight

(Since you type the answers in WebAssign in little boxes, you can forget to learn the forms that answers should take. These problems should all be really easy: I want to make sure you know how to write the answers correctly.)

(S1) Compute $\langle 1,2,3\rangle+\langle 2,-2,2\rangle$. Write the answer both in vector notation and in i, j, k notation.
(S2) Compute $\langle 1,2,3\rangle\cdot\langle 2,-2,2\rangle$.
(S3) Compute $\langle 1,1,0\rangle\times\langle 1,0,1\rangle$.
(S4) Write an implicit equation for the plane containing the points $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.
(S5) Write a parametric equation for the line through $(1,0,0)$ in the direction $\langle 1,1,1\rangle$.
(S6) Write implicit equations for the the line through $(1,0,0)$ in the direction $\langle 1,1,1\rangle$.

2. Projection and Change of Basis

Remember that an *orthonormal basis* for $\mathbb{R}^2$ is a pair of vectors $\vec{e}_1, \vec{e}_2$ so that $\|\vec{e}_1\| = \|\vec{e}_2\| = 1$ and $\vec{e}_1\cdot\vec{e}_2 = 0$.

(1) Suppose $\vec{e}_1 = \langle 3/5, 4/5\rangle$. Find a second vector $\vec{e}_2$ so that $\vec{e}_1, \vec{e}_2$ form an orthonormal basis.
(2) Consider the vector $\vec{v} = \langle 2,4\rangle$.
  (a) Compute $\mathrm{comp}_{\vec{e}_1}(\vec{v})$ and $\mathrm{comp}_{\vec{e}_2}(\vec{v})$.
  (b) Verify that $\vec{v} = \mathrm{comp}_{\vec{e}_1}(\vec{v})\vec{e}_1 + \mathrm{comp}_{\vec{e}_2}(\vec{v})\vec{e}_2$.
(3) Show that for *any* vector $\vec{v} = \langle x, y\rangle$, $\vec{v} = \mathrm{comp}_{\vec{e}_1}(\vec{v})\vec{e}_1 + \mathrm{comp}_{\vec{e}_2}(\vec{v})\vec{e}_2$ (by computing $\mathrm{comp}_{\vec{e}_1}(\vec{v})$ and $\mathrm{comp}_{\vec{e}_2}(\vec{v})$).
(4) Find an orthonormal basis $\vec{e}_1, \vec{e}_2, \vec{e}_3$ for $\mathbb{R}^3$ so that $\vec{e}_1$ and $\vec{e}_2$ lie in the plane $x+y+z=0$ (and, consequently, $\vec{e}_3$ is perpendicular to this plane). If the computations get tedious, feel free to use Mathematica to assist—print out the Mathematica code you use and turn it in with your assignment. If you use Mathematica to help with the computation, make sure to also use it to check your answers.

(The fact that projections let you write a vector in terms of any orthonormal basis is *the most important* use of projections, though force diagrams (and other physical applications) and computer graphics are other common uses. This technique works in any dimension.)

3. Mathematica

Mathematica has different commands for plotting parametric curves / surfaces defined by parametric and implicit equations, in 2 and 3 dimensions.

(1) You can compute cross products with the `Cross` function. For example,
`Cross[{1,2,3},{4,5,6}]` or `Cross[{1,0,0},{0,1,0}]`
Try it.

(2) To plot the line $\langle x, y\rangle = \langle 1 + 2t, 3 + 4t\rangle$ (for instance), in the range $-2 \leq t \leq 2$, use
`ParametricPlot[{1 + 2*t, 3 + 4*t}, {t, -2, 2}]`
Remember that Mathematica uses { and } for vectors, instead of ⟨ and ⟩. The `{t, -2, 2}` indicates the range of t values.
Try it.

(3) You can plot several curves at once by giving them as a list, such as
`ParametricPlot[{{1 + 2*t, 3 + 4*t}, {t + 1, t + 2}}, {t, -2, 2}]`
Try it.

(4) In 3 dimensions, the analogous command is `ParametricPlot3D`
`ParametricPlot3D[{t, t + 2, 2 t + 3}, {t, -2, 2}]`
Try it.

(5) An implicit equation for the first line is $2x - y = -1$. You can plot an implicit equation with `ContourPlot`. Here, you have to specify both the x- and y-ranges, and use `==` for equality. (As in most programming languages, there is a difference between testing for equality and defining a variable; `==` is used to test if two things are equal; try `2==3` and see what happens.)
`ContourPlot[2*x - y == -1, {x, -1, 1}, {y, -1, 2}]`
It can take some playing around to get a good window.
Try it.

(6) The 3D version, useful for planes, is `ContourPlot3D`. For instance, the plane $x+2y+3z = 4$:
`ContourPlot3D[x + 2 y + 3 z == 4, {x, -1, 1}, {y, 0, 1}, {z, 0, 1}]`
The plane $x+y+z = 1$: `ContourPlot3D[x + y + z == 1, {x, 0, .8}, {y, 0, .8}, {z, 0, .8}]`
Try both.

(7) You can also plot parametric equations for planes, using `ParametricPlot3D`. For example, the plane $x + y + z + 1$ is given by $\langle x, y, z\rangle = \langle 1 + s + t, -s, -t\rangle$. You can plot this plane parametrically with
`ParametricPlot3D[{s + t + 1, -s, -t}, {s, -1, 1}, {t, -1, 1}]`
Try it.

(8) Spheres are convenient to plot with `ContourPlot3D`. Plot a sphere centered at $(1, 2, 3)$ of radius .25. (Spheres will be a convenient way to represent points.)

(9) You can combine several plots into one by assigning them to variables and then using `Show`. Here's a demonstration that the plane through $\langle 1, 1, 1\rangle$, $\langle 1, 2, 0\rangle$, and $\langle 2, 2, 1\rangle$ is given by $\langle -1, 1, 1\rangle \cdot \langle x, y, z\rangle = \langle -1, 1, 1\rangle \cdot \langle 1, 1, 0\rangle$:
```
planeplot = ContourPlot3D[{-1, 1, 1}.{x, y, z} == {-1, 1, 1}.{1, 1, 1},
 {x, 0, 3}, {y, 0, 3}, {z, 0, 3}];
point1 = ContourPlot3D[(x - 1)^2 + (y - 1)^2 + (z - 1)^2 == .05, {x, 0, 3},
 {y, 0, 3}, {z, 0, 3}];
point2 = ContourPlot3D[(x - 1)^2 + (y - 2)^2 + z^2 == .05, {x, 0, 3},
 {y, 0, 3}, {z, 0, 3}];
point3 = ContourPlot3D[(x - 2)^2 + (y - 2)^2 + (z - 1)^2 == .05, {x, 0, 3},
 {y, 0, 3}, {z, 0, 3}];
Show[{planeplot, point1, point2, point3}, PlotRange -> All]
```
Try it. (The semicolons at the ends of the first three lines tell Mathematica not to show the output of those lines. If you're confused by what's going on here, try removing those semicolons, and try running just `Show[point1]`. The `PlotRange -> All` is so that if the images have different bounding boxes, you see all of them.)

(10) Question 7 on your WebAssign homework due today was to check if a certain pair of lines were parallel, skew, or intersecting. Use Mathematica to plot them both in a single plot as evidence for your answer. (Which is more convincing: the plot or the computation?)

(11) Question 11 on your WebAssign homework was to find an equation for a plane through a particular point containing a particular line. Plot the plane, line, and point in a single graph, to check your work.

(12) **Optional.** Check question 13 on the WebAssign homework.

(13) **Optional.** Write a Mathematica function which takes as input three (non-colinear) points and returns a ContourPlot3D of the plane containing those three points.

(14) Clean up your worksheet (deleting code which didn't work), print it out, and turn it in with your assignment.

We didn't discuss how to plot the implicit equations for a line (e.g., what the book calls "symmetric equations"), in 3 dimensions. I am not aware of a natural way to do this in Mathematica, though perhaps there is one. (Computationally, it's a fairly different problem than implicitly plotting planes... why?)

E-mail address: `lipshitz@math.columbia.edu`