Visualizing implicitly defined surfaces

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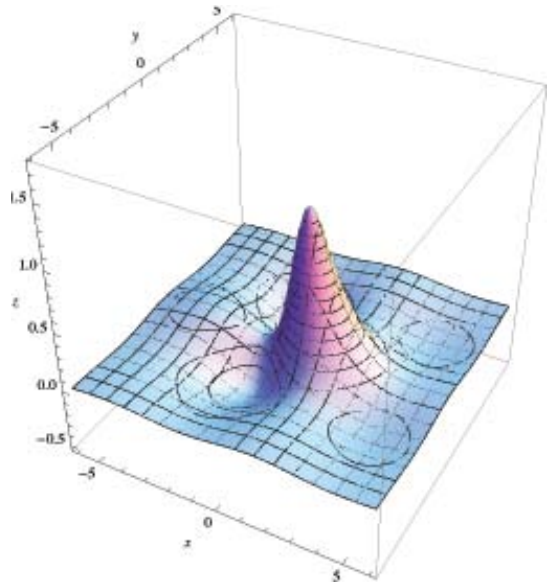
September 21, 2009

Warm-up example

This is a plot of the surface

$$z = (\sin(x) + \cos(y))/(1 + x^2 + y^2)$$

```
ContourPlot3D[(Sin[x]+Cos[y])/(1+x^2+y^2)-z==0,{x,-6,6},{y,-6,6},{z,-.5,1.75},
AxesLabel-> {x,y,z}]
```

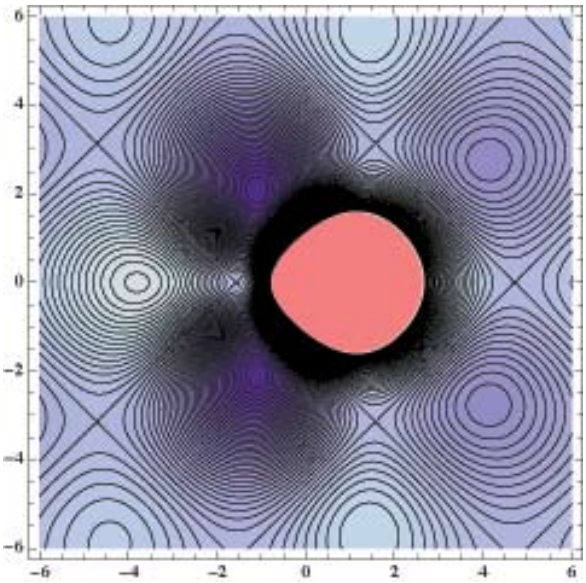


We can pass a plane $z = \text{const}$ hrough this and look at the curves that we get. These are called traces. You can animate them with this command (output not shown):

```
Animate[ContourPlot[(Sin[x]+Cos[y])/(1+x^2+y^2)-z==0,{x,-6,6},{y,-6,6},
  PlotPoints->40,PlotRange->{-6,6}],{z,-.5,1.25,.0025}]
```

We can also draw all of those contours at once:

```
ContourPlot[(Sin[x]+Cos[y])/(1+x^2+y^2),{x,-6,6},{y,-6,6},Contours->60,
  PlotPoints->60,ClippingStyle->Pink]
```

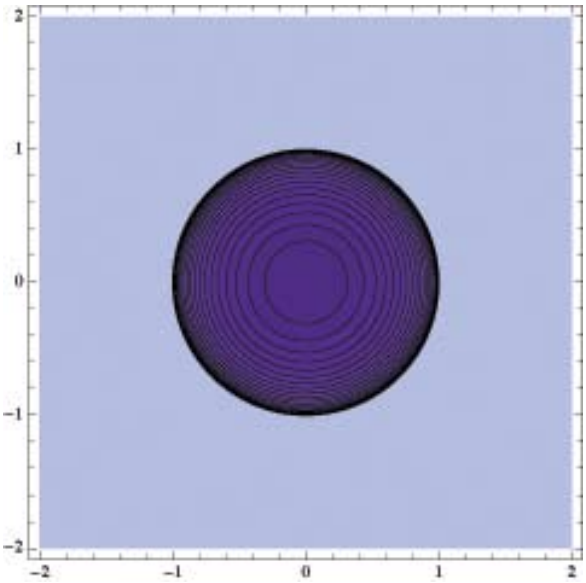


Quadrics

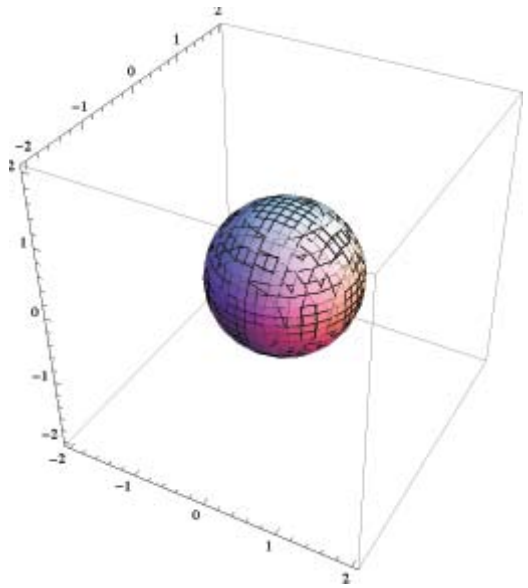
Here are the plots of the "Standard models" and their traces:

First: $x^2 + y^2 + z^2 = 1$.

```
ContourPlot[x^2+y^2-1,{x,-2,2},{y,-2,2},PlotPoints->40,ClippingStyle->Pink,
  Contours->Function[-Range[-2,0,.05]^2]]
```

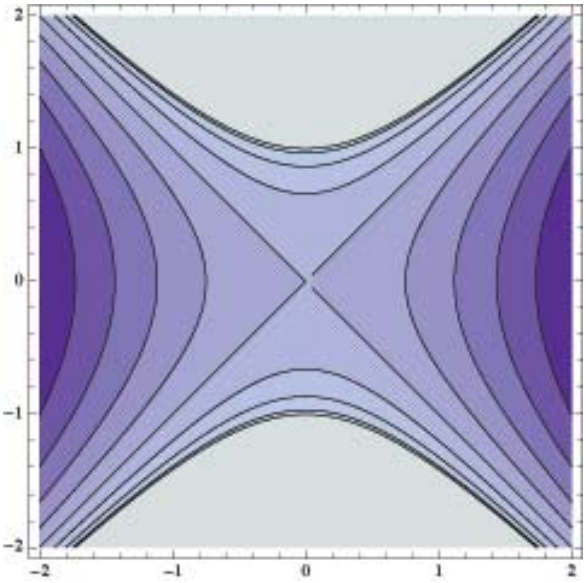



```
ContourPlot3D[x^2+y^2+z^2==1,{x,-2,2},{y,-2,2},{z,-2,2}]
```

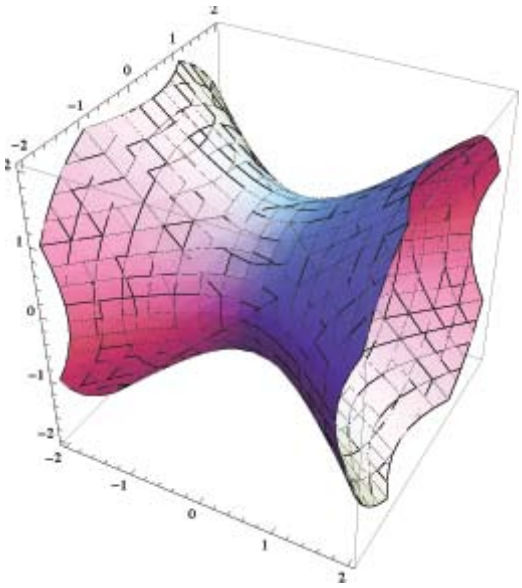


Next: $-x^2 + y^2 + z^2 = 1$.

```
ContourPlot[-x^2+y^2-1,{x,-2,2},{y,-2,2},PlotPoints->40,ClippingStyle->Pink,
  Contours->Function[-Range[-4,0,.25]^2]]
```

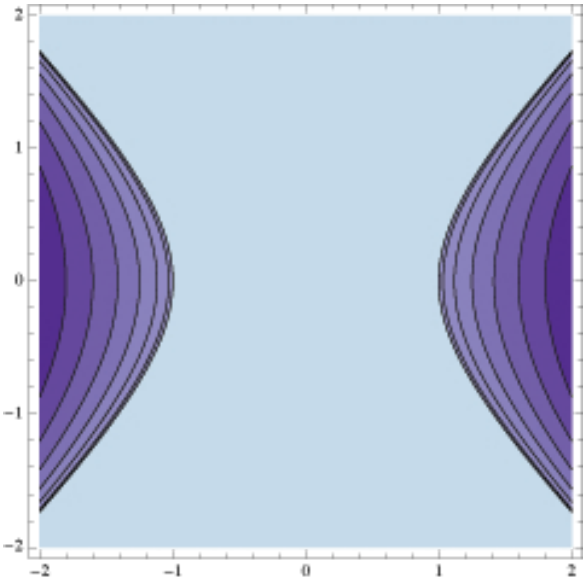



```
ContourPlot3D[-x^2+y^2+z^2==1,{x,-2,2},{y,-2,2},{z,-2,2}]
```

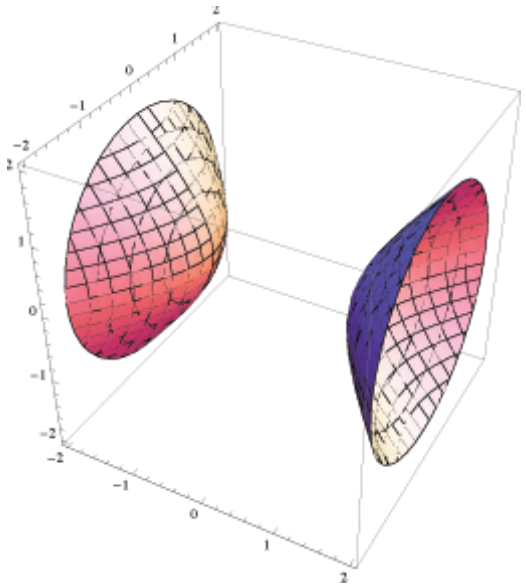


Next: $-x^2 + y^2 + z^2 = -1$.

```
ContourPlot[-x^2+y^2+1,{x,-2,2},{y,-2,2},PlotPoints->40,ClippingStyle->Pink,
  Contours->Function[-Range[-4,0,.25]^2]]
```

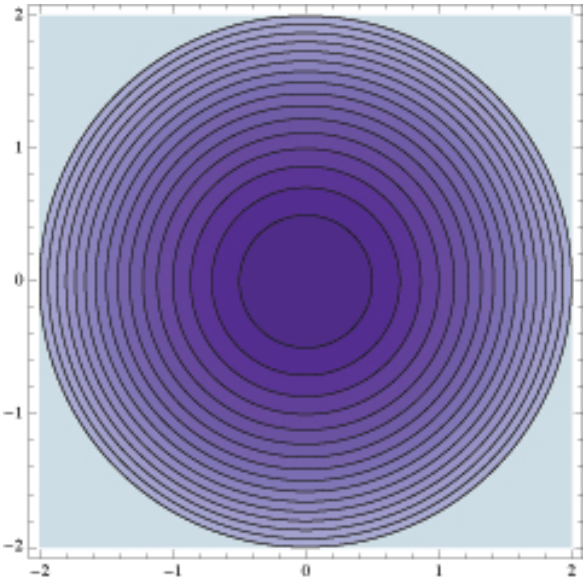



```
ContourPlot3D[-x^2+y^2+z^2==-1,{x,-2,2},{y,-2,2},{z,-2,2}]
```

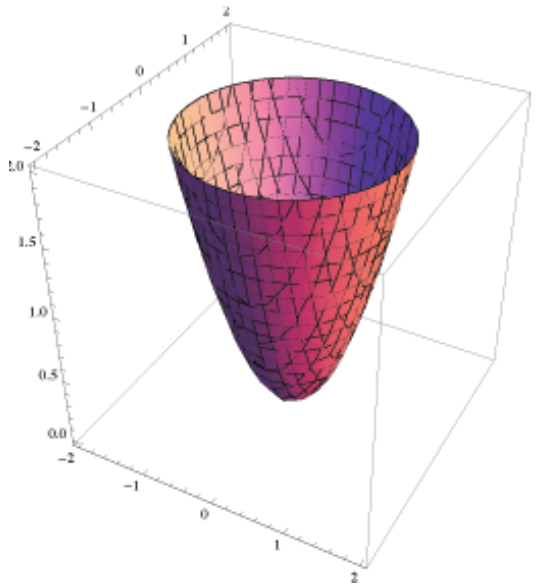


Next: $x^2 + y^2 - z = 0$.

```
ContourPlot[x^2+y^2,{x,-2,2},{y,-2,2},PlotPoints->40,ClippingStyle->Pink,
  Contours->Function[Range[0,4,.25]]]
```

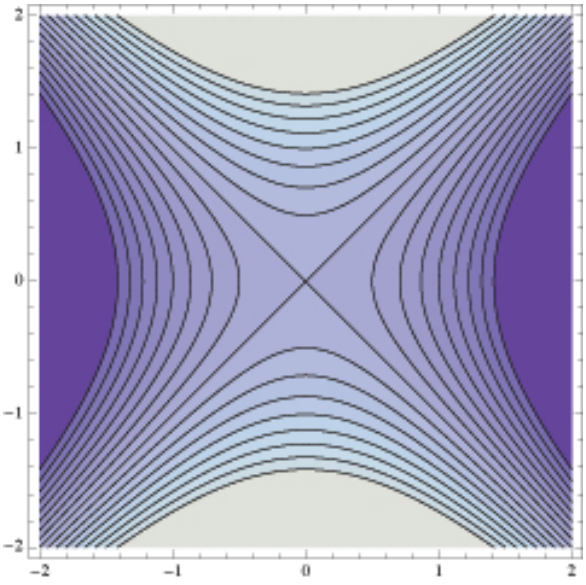



```
ContourPlot3D[z-x^2-y^2==0,{x,-2,2},{y,-2,2},{z,0,2}]
```

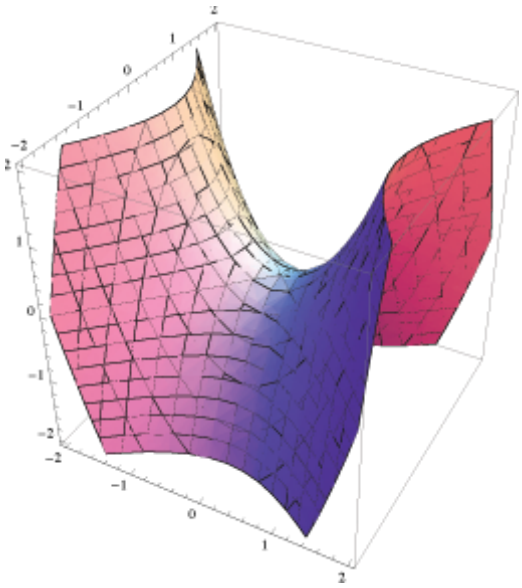


Next: $x^2 - y^2 - z = 0$.

```
ContourPlot[-x^2+y^2,{x,-2,2},{y,-2,2},PlotPoints->40,ClippingStyle->Pink,
  Contours->Function[Range[-2,2,.25]]]
```




```
ContourPlot3D[z-x^2+y^2==0,{x,-2,2},{y,-2,2},{z,-2,2}]
```



Here's a sphere (orange) and a more general ellipsoid (blue):

```
ContourPlot3D[{x^2+y^2+z^2==1,9*x^2+4*y^2+z^2==3},{x,-1.75,1.75},{y,-1.75,1.75},
  {z,-1.75,1.75},Mesh->None,ContourStyle->{Directive[Orange,
  Opacity[0.8],Specularity[White,30]],Directive[Blue,Opacity[0.8],
  Specularity[White,30]]},PlotPoints->30]
```