

Topology, fall 2022

Homework 8, due Friday November 11. (60 points)

Read Munkres Sections §52, 53, 54.

I, II. Exercises 3,4 on page 341.

III. (a) Prove that $\pi_1(X \times Y, x_0 \times y_0) \cong \pi_1(X, x_0) \times \pi_1(Y, y_0)$ (fundamental group of the direct product is the product of fundamental groups).

(b) Use the above result to compute π_1 of the 2-torus $\mathbb{S}^1 \times \mathbb{S}^1$.

VI. Prove that if X is contractible then $\pi_1(X, x_0)$ is trivial. Use the homotopy from the identity map on X to the map to a point in X . What happens to the loop representing an element of $\pi_1(X, x_0)$ under this homotopy? Note that the homotopy may move the basepoint x_0 , and your proof needs to account for that.

V. Exercise 5 on page 348.

VI. Exercise 6 on page 348.