Topology, fall 2022

Homework 1, due Thursday Sept 15 (online submission via Courseworks).

Read Munkres sections §12 – §16 (pages 75 - 92). Write solutions to the following problems (some of these problems are exercises in §13).

1. Let X be a topological space and A a subset of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.

2. Example 1 of §12 page 76 lists 9 topologies on a 3-element set $X = \{a, b, c\}$. Label these topologies $T^1, T^2, \ldots, T^9$ from left to right and top to bottom.
(a) Which of these is the discrete topology and which is indiscrete?
(b) Which of these topologies restrict to the discrete topology on the subspace $Y = \{a, c\}$? (In class we discussed restricting topology to a subspace; also see §16.)
(c) Among these nine, select four topologies F_1, F_2, F_3, F_4 so that F_{i+1} is strictly finer than F_i for each $i = 1, 2, 3$.
(d) Which of the nine topologies admit a basis with exactly two sets? Which of the nine admit a subbasis with exactly two sets? (For this problem assume that a basis or a subbasis does not contain the empty set.)

3. Describe all possible topologies on a two-element set. How many are there?

4. (This is problem 8a in Munkres §13). Show that the countable collection

$$\mathcal{B} \; = \; \{ \; (a, b) \mid a < b, \; a \text{ and } b \text{ are rational } \}$$

is a basis that generates the standard topology on $\mathbb{R}$.

Write solutions to problems 2, 3, 4 on pages 91-92 in Munkres (end of Section 16).