Topology, fall 2015, Practice Quiz

1. Mark the boxes that are followed by correct statements.

☐ Any compact space is metrizable.

☐ Any topology on a 3-element set is Hausdorff.

☐ The only homeomorphism from the Cantor set to itself is the identity.

☐ Any subspace of a Hausdorff space is Hausdorff.

☐ Any metrizable space is second-countable.

☐ The space $[0,1]^\omega$ with the product topology is metrizable.

☐ Any finite T_1-space is discrete.

☐ Direct product of two metrizable spaces is metrizable.

☐ If $A, B \subset X$ are both compact then $A \cup B$ is compact.

☐ A Cantor set C is homeomorphic to a disjoint union $C \sqcup C$ of two copies of itself.

☐ The set $\mathbb{N}$ of natural numbers equipped with the discrete topology is metrizable.

☐ In a finite complement topology, any sequence has a convergent subsequence.

☐ Any dense subset of the Cantor set is uncountable.

☐ A subspace of $\mathbb{R}$ is connected if and only if it is path-connected.

☐ If $A, B \subset X$ are path-connected then $A \cup B$ is path-connected.

☐ Direct product $X \times Y$ of connected spaces X, Y is connected.

☐ $A \subset X$ is connected if and only if $\overline{A}$ is connected.

☐ $\mathbb{N}$, in the finite complement topology, is second-countable.

☐ For any metric space X, map $f : X \times X \times X \longrightarrow \mathbb{R}$ that takes (x, y, z) to

$$f(x, y, z) = d(x, y)d(y, z) - d(x, z)$$

is continuous.

2. Which of the following spaces are compact?

The Cantor set C.

Intersection of the Topologist Sine Curve in $\mathbb{R}^2$ with the closed unit disk.

$\mathbb{Q}$, with the basis of open sets $\{(-\infty, a) | a \in \mathbb{Q}\}$.

Subspace $[0, 1]$ of $\mathbb{R}_\ell$.

Intersection $[0, 1] \cap \mathbb{Q}$, in the standard topology.