Topology, fall 2015, Review problems

The review session is on Friday, December 18, 5:30-7pm, in room 507. We will also discuss some homework problems, especially from the last two homeworks.

1. Mark the boxes that are followed by correct statements.

☐ Any covering $E \longrightarrow S^1$ of a circle is connected.

☐ Any map from the Möbius band to the circle is null-homotopic.

☐ Möbius band is compact.

☐ Any connected subspace of $\mathbb{R}$ has the trivial fundamental group.

☐ Any compact subspace of the Klein bottle is path-connected.

☐ There exists a surface with the fundamental group $\mathbb{Z}/3$.

☐ Any deformation retract of a compact space is compact.

☐ Any continuous map from the theta-space to itself has a fixed point.

☐ Space $\mathbb{R}_\ell$ has trivial fundamental group (pick a base point on $\mathbb{R}_\ell$ for this question).

2. (a) Give the definition of a linear continuum.

(b) Can a linear continuum be connected but not simply-connected in the order topology?

3. (a) A space is called *totally disconnected* if its only connected subspaces are one-point sets. Prove that the Cantor set is totally disconnected.

(b) Is Cantor set metrizable?

4. (a) Give an example of a Hausdorff topological space X and an equivalence relation $\sim$ on it so that $Y = X/\sim$ is not Hausdorff. In your example, is Y a T_1-space? Is it compact? Can you give an example with a simply-connected Y?

(b) Give an example of a Hausdorff space X and a non-T_1 space Y which are homotopy equivalent.

5. Determine fundamental groups of the following spaces:

(a) Two Möbius bands with their boundary circles identified.

(b) Two Möbius bands with their core circles identified.

(c) A bouquiet of a degree three dunce cap and a projective plane.

(d) $\mathbb{RP}^3$ with two punctures.

(e) $\mathbb{R}^2$ with three disjoint closed intervals on the x-axis deleted.

(f) $\mathbb{R}^3$ with the x, y and z-axes deleted.

6. Determine which surfaces result from the following gluing schemes:

(a) abc, abc (b) abc, acb (c) $abc, a\bar{b}\bar{c}$ (d) $abcd, dc\bar{a}b$

7. (a) Show that the 2-torus admits a self-covering $T^2 \longrightarrow T^2$ of any degree.

(b) Show that the 2-torus T^2 is a degree 2 covering space of the Klein bottle KB.

8. The attached sheet is the printout of page 58 from A.Hatcher's book "Algebraic Topology". Among coverings (8)-(14) of the figure eight, single out those that are regular. Which of these coverings are finite? Which of them have abelian fundamental

group?

9. Connected covering $E \longrightarrow B$ has degree four, where B is a bouquiet of three circles. What can you say about the fundamental group of E?

Additional problems from Munkres that cover the last lecture: page 483 exercises 1, 2, 5(optional).