Topology, fall 2015

Homework 4, due Wednesday October 6 before class.

Read Sections 21, 23, 24, 25 (up to page 160 inclusive).

Write solutions to exercises

I. Let (X, d) be a metric space and $f : X \longrightarrow X$ a continuous map. Prove that the function $g : X \longrightarrow \mathbb{R}$ defined by $g(x) = d(x, f(x))$ is a continuous function. (Hint: factor g into a composition of continuous functions.)

II. (a) Show that $(-\infty, a) \cup [a, +\infty)$ is a separation of the space $\mathbb{R}_\ell$ for any real a. (b) Describe connected components of $\mathbb{R}_\ell$ and classify all continuous maps $\mathbb{R} \longrightarrow \mathbb{R}_\ell$.

Munkres exercise 3 on page 152.

Exercises on pages 157-158:

- Exercise 1 (imbeddings are defined on page 105)
- Exercise 2 (Hint: first show that $g(x) = f(x) - f(-x)$ is a continuous function. What can you say about the range of $g(x)$ if there is no x with $f(x) = f(-x)$? What happens to $g(x)$ as you travel half-way around the circle?)
- Exercises 5, 8 (give brief justifications for your answers).

III. Classify letters of the English alphabet by their homeomorphism type.

Additional suggested problems to think through: 5, 6 on page 127, 10 on page 152, 4, 9, 11 on page 158, 2 on page 162.