Topology, fall 2015

Homework 11, due Wednesday December 9 before class.

Read §70 − 73 (in §71 you can read up to and including Theorem 71.3, the rest of §71 is optional reading).

I. Exercise 3ab on page 433.

II. Give an example where the Seifert-van Kampen theorem fails if one does not assume that $U \cap V$ is path-connected.

III. (a) What is the relation between punctured $\mathbb{RP}^2$ (that is, $\mathbb{RP}^2$ with a point deleted) and the Möbius band?
(b) Remove two points from $\mathbb{RP}^2$ to get a space $X = \mathbb{RP}^2 \setminus \{p, q\}$. Determine the fundamental group of X. (Hint: if you solve (a) first, you'll be able make a picture of X. Then look for a homotopy retract of X with the fundamental group easy to determine).

IV. Find fundamental groups of the following spaces:
(a) Wedge $\mathbb{RP}^2 \vee \mathbb{RP}^2$ of two projective planes.
(b) Wedge $\mathbb{RP}^2 \vee S^1 \vee S^2$.
(c) Once punctured Klein bottle $KB \setminus \{p\}$.
(d) Twice punctured Klein bottle $KB \setminus \{p, q\}$.
(e) Once punctured oriented surface M of genus 2.
(f) Two-sphere with a disc B^2 attached along the equator.
(g) Take two two-dimensional tori T_1^2 and T_2^2 and identify a longitudinal circle of the first tori with a meridianal circle of the second tori. Determine π_1 of the resulting space.

V. Take a solid torus $B^2 \times S^1$, a Hausdorff space Y, and a continuous map $\psi : T^2 \longrightarrow Y$ from the boundary surface $T^2 = S^1 \times S^1$ of the solid torus. Glue the solid torus and Y via this map (by identifying point $s \in T^2$ with its image in Y for all s) and denote the resulting space X. Explain the relation between the fundamental groups of Y and X by analogy with the Theorem 72.1 (also discussed in class).