Topology, fall 2015

Homework 10, due Wednesday December 2 before class.

Read §59 – 60 and §67 – 69.

I. Compute fundamental groups of the following spaces:
(a)$S^1 \times S^2$.
(b) A bouquet of two two-spheres $S^2 \vee S^2$. (Hint: Find a covering by two open sets so you can apply Corollary 59.2. Can you thicken each S^2 in the bouquet to an open set without changing its homotopy type?) Does your proof generalize to $S^n \vee S^m$ for all $n, m > 2$?
(c) A bouquet of 1-sphere and 2-sphere $S^1 \vee S^2$. You'll need to use theorem 59.1 to show that π_1 is not too large. To show that it's not too small, prove that the inclusion of S^1 into this bouquet induces an injective map of fundamental groups.
(d) $\mathbb{R}^3 \setminus \{p, q\}$, that is, $\mathbb{R}^3$ without two points p, q. Hint: find a deformation retraction onto a much smaller space. Briefly explain or draw the retraction, don't write explicit formulas. You can also first recall how the retraction works for the space $\mathbb{R}^3 \setminus \{p\}$.

II. In class we defined the projective space $\mathbb{RP}^n$. Consider $n = 1$ space. Identify $\mathbb{RP}^1$ with a more familiar space and describe explicitly the covering map $S^1 \longrightarrow \mathbb{RP}^1$.

III. Exercise 4 on page 412 and its extension:
(d) Determine torsion subgroups of the following abelian groups: $\mathbb{Q}$, $\mathbb{Q}/\mathbb{Z}$, $\mathbb{R}/\mathbb{Z}$. Here $\mathbb{Q}$, $\mathbb{R}$ are considered as groups under the addition operation.

IV. Exercise 3 on page 421.

V. Which of the following groups given by relations and generators are trivial? Which of them can you explicitly identify, for instance as cyclic groups of a particular order, etc.?
(a) $(g|g^{10}, g^{13})$,
(b) $(g|g^{10}, g^{14})$,
(c) $(a, b|aba, abab)$,
(d) $(a, b|a^2, ab^2a^{-1})$,
(e) $(a, b|a^2b^3, ab^2)$.