Topology, fall 2015

Homework 1, due Wednesday September 16 before class.

Read Sections §12, §13 in Munkres (pages 75 - 83). Write solutions to the following problems (some of these problems are exercises in §13).

1. Let X be a topological space and A a subset of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.

2. Example 1 of §12 page 76 lists 9 topologies on a 3-element set $X = \{a, b, c\}$. Label these topologies $T^1, T^2, \ldots, T^9$ from left to right and top to bottom.
(a) Which of these is the discrete topology and which is indiscrete?
(b) Which of these topologies restrict to the discrete topology on the subspace $Y = \{a, c\}$? (In class we briefly discussed restricting topology to a subspace; also see the beginning of §16.)
(c) Among these nine, select four topologies F_1, F_2, F_3, F_4 so that F_{i+1} is strictly finer than F_i for each $i = 1, 2, 3$.
(d) Which of the nine topologies admit a basis with exactly two sets? Which of the nine admit a subbasis with exactly two sets? (For this problem assume that a basis or a subbasis does not contain the empty set.)

3. Describe all possible topologies on a two-element set. How many are there?

4. Show that topologies $\mathbb{R}_\ell$ and $\mathbb{R}_K$ are not comparable. These topologies were defined in class and also discussed in Munkres §13 pages 81-82.

5. (This is problem 8a in Munkres §13). Show that the countable collection

$$\mathcal{B} \ = \ \{ \ (a, b) \mid a < b, \ a \text{ and } b \text{ arerational } \}$$

is a basis that generates the standard topology on $\mathbb{R}$.