Modern algebra I, spring 2017. Quiz 5

Name:_____ UNI: _____

Check the boxes that are followed by correct statements.

☒ Any two orbits of a group G acting on a set X are either disjoint or equal.

TRUE

☐ The centralizer of the permutation (123) in S_5 has order 3.

FALSE. The centralizer is generated by powers of $g=(123)$ and by permutations of 4,5. The centralizer has order 6.

☒ There is a group of order 50 that has a normal Sylow 2-subgroup.

TRUE. Take an abelian group of order 50. Any subgroup is normal

☒ 3-cycle (234) generates a Sylow 3-subgroup of S_4.

TRUE. $|S_4| = 4! = 4\cdot 3\cdot 2\cdot 1 = 2^3 3 \Rightarrow$ A sylow 3-subgroup has order 3. 3-cycle (234) generates a subgroup of order 3.

☒ Dihedral group D_6 is solvable.

TRUE. Use the chain of groups $D_6 \supset \mathbb{C}_6 \supset \{1\}$

☐ Any Sylow 2-subgroup of S_4 is abelian.

FALSE. We checked that D_4 is a Sylow 2-subgroup of S_4. D_4 is not abelian.