Modern algebra I, spring 2017. Quiz 4

Name:_____ UNI: _____

Check the boxes that are followed by correct statements.

☑ The regular action of a group G on itself has only one orbit.

The regular action is the action of G on itself by left multiplication. We can go from $1 \in G$ to any $g \in G$ by applying $g(1) = g1 = g$. There is only one orbit.

☑ If a group G of order six acts transitively on a three-element set, then the stabilizer of any point has order two.

Transitive action is an action with only one orbit. Stabilizers of points on an orbit are conjugate subgroups and have the same cardinality $|G| = |\mathcal{O}_x| \cdot |G_x|$. Since $|G| = 6$, $|\mathcal{O}_x| = 3 \Rightarrow |G_x| = 2$. True

☐ The conjugation action of the symmetric group S_3 on itself has exactly two orbits. False. ~~The sum~~ Orbits of the conjugation action of G on itself are conjugacy classes. S_3 has three conjugacy classes, not two: identity, transpositions & 3-cycles.

☑ The permutation action of the symmetric group S_3 on $\{1, 2, 3\}$ has no fixed points. A fixed point $x \in X$ of an action of G is a point such that $gx = x$ for all $g \in G$. But already the 3-cycle $(1,2,3)$ has no fixed points acting on $\{1,2,3\}$

☑ Any Sylow 2-subgroup of S_3 is abelian.

A Sylow 2-subgroup of S_3 has order 2, since $|S_3| = 6 = 2 \cdot 3$. Any group of order 2 is abelian

☑ The subgroup C_5 of all rotations is a Sylow 5-subgroup of the dihedral group D_5.

C_5 has order 5 and $|D_5| = 10 = 5 \cdot 2$

Note that C_5 is normal, and thus the only Sylow 5-subgroup of D_5.