Homework 1

1.1. #6

$$\left|\begin{array}{l} x+2y+3z=8 \\ x+3y+3z=10 \\ x+2y+4z=9 \end{array}\right| \begin{array}{l} \\ -(\mathrm{I}) \\ -(\mathrm{I}) \end{array} \longrightarrow \left|\begin{array}{l} x+2y+3z=8 \\ \quad\; y \qquad\quad =2 \\ \qquad\qquad z=1 \end{array}\right| \xrightarrow{-3(\mathrm{III})}$$

$$\rightarrow \left|\begin{array}{l} x+2y \qquad =5 \\ \quad\; y \qquad\; =2 \\ \qquad\quad z=1 \end{array}\right| \xrightarrow{-2(\mathrm{II})} \left|\begin{array}{l} x \qquad\qquad =1 \\ \quad\; y \qquad\; =2 \\ \qquad\quad z=1 \end{array}\right|$$

The system has unique solution $x=1, y=2, z=1$

#17

$$\left|\begin{array}{l} x+2y=a \\ 3x+5y=b \end{array}\right|$$

Convert to augmented matrix

$$\left(\begin{array}{cc|c} 1 & 2 & a \\ 3 & 5 & b \end{array}\right) \xrightarrow[-3(\mathrm{I})]{} \left(\begin{array}{cc|c} 1 & 2 & a \\ & -1 & b-3a \end{array}\right) \xrightarrow[\times(-1)]{}$$

$$a-2(3a-b)= \\ =2b-5a$$

$$\left(\begin{array}{cc|c} 1 & 2 & a \\ 0 & 1 & 3a-b \end{array}\right) \xrightarrow{-2(\mathrm{II})} \left(\begin{array}{cc|c} 1 & 0 & 2b-5a \\ 0 & 1 & 3a-b \end{array}\right)$$

For all a and b

The system has a unique solution $\begin{cases} x=2b-5a \\ y=3a-b \end{cases}$

1.2 #4 $\left|\begin{array}{l} x+y=1 \\ 2x-y=5 \\ 3x+4y=2 \end{array}\right| \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -1 & 5 \\ 3 & 4 & 2 \end{array}\right) \xrightarrow[\substack{-2(\mathrm{I}) \\ -3(\mathrm{I})}]{} \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 1 & -1 \end{array}\right) \xrightarrow[/(-3)]{}$

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{array}\right) \xrightarrow[-(\mathrm{II})]{-(\mathrm{II})} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right)$$

there is a unique solution $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

#6 $\left|\begin{array}{l} x_1 - 7x_2 + x_5 = 3 \\ x_3 - 2x_5 = 2 \\ x_4 + x_5 = 1 \end{array}\right|$

$$\downarrow \quad \left(\begin{array}{ccccc|c} 1 & -7 & 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right)$$

The systems is already in the RREF form.

the pivots are in 1st, 3rd, 4th columns. these are the leading variables. No pivots in columns 2, 5 so x_2, x_5 are free variables.

$$\begin{cases} x_1 = 7x_2 - x_5 + 3 \\ x_3 = 2x_5 + 2 \\ x_4 = -x_5 + 1 \end{cases} \xrightarrow[]{\substack{x_2 = t \\ x_5 = r}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 7t - r + 3 \\ t \\ 2r + 2 \\ 1 - r \\ r \end{pmatrix}$$

1.2 #8 $\left| \begin{array}{r} x_2 + 2x_4 + 3x_5 = 0 \\ 4x_4 + 8x_5 = 0 \end{array} \right| \longrightarrow \left(\begin{array}{ccccc|c} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 4 & 8 & 0 \end{array} \right) \xrightarrow[\div 4]{}$

$$\left(\begin{array}{ccccc|c} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array} \right) \xrightarrow{-2(\mathrm{II})} \left(\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array} \right)$$

$$\left(\begin{array}{l} x_2 = x_5 \\ x_4 = -2x_5 \end{array} \right) \xrightarrow{x_1 = t,\ x_3 = r,\ x_5 = s} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} t \\ s \\ r \\ -2s \\ s \end{bmatrix} \qquad t, r, s \in \mathbb{R}$$

x_1, x_3, x_5 are free variables

x_2, x_4 are leading variables

there is a 3-parameter family of solutions.