Lie groups

Homework #4, due Wednesday, October 22.

1. Find multiplicities of irreducible $sl(2)$ representations in $V_1^{\otimes 4}$, $V_2^{\otimes 3}$, $V_1 \otimes V_3 \otimes V_5$.

2. Determine how $S^2(V_n)$ and $\Lambda^2(V_n)$ decompose into irreducibles (start with small n and use characters). Do the same for $\Lambda^2(V_n \oplus V_m)$ (hint: how can you simplify $\Lambda^2(V \oplus W)$?)

3. Compute the Killing form on the Lie algebra of upper-triangular 2×2 matrices. What's the kernel of this form?

4.Prove: If a Lie algebra is nilpotent, its Killing form is identically 0.

5. We proved that if a complex finite-dimensional Lie algebra is solvable then every irreducible representation is one-dimensional. Show the converse, by looking at the Jordan-Hölder filtration of the adjoint representation.

6*. Prove that the multiplicity of V_0 in $V_1^{\otimes 2n}$ is the n-th Catalan number $\frac{1}{n+1}\begin{pmatrix} 2n \\ n \end{pmatrix}$.