Lie groups

Homework #3, due Wednesday, October 8.

1. Prove that L is solvable if and only if there exists a chain of subalgebras

$$0 = L_0 \subset L_1 \subset \cdots \subset L_{n-1} \subset L_n = L$$

such that L_i is an ideal of L_{i+1} and the quotient L_{i+1}/L_i is abelian for each i.

2. Prove that the center of the Lie algebra $\mathfrak{gl}(n, \Bbbk)$ is one-dimensional.

3. Let $L = \mathfrak{sl}(n, \Bbbk)$. Show that $[L, L] = L$.

4. Classify all ideals of the Heisenberg Lie algebra (3-dimensional Lie algebra with a basis $\{x, y, z\}$ such that $[x, y] = z$ and z is central, $[z, x] = [z, y] = 0$). Is this algebra nilpotent? Solvable? Simple?

5. Prove that L is simple if $\dim(L) = 3$ and $L = [L, L]$.

6. Let L be a solvable Lie algebra. Show that the adjoint representation of L is completely reducible if and only if L is abelian.