Lie groups and representations, Fall 2009

Homework #7, due Monday, November 23.

1. Determine the terms of degrees 3 and 4 in the Baker-Campbell-Hausdorff formula (we derived in class that $x * y = x + y + \frac{1}{2}[x, y] + \ldots$).

2. Check that the Lie groups

$$\begin{aligned} G_1 &= \{\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}, \quad a, b \in \mathbb{R}, a > 0\} \\ G_2 &= \{\begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix}, \quad a, b \in \mathbb{R}, a > 0\} \end{aligned}$$

have isomorphic Lie algebras. Find an explicit isomorphism $G_1 \cong G_2$.

3. Let L be the 2-dimensional Lie algebra with basis $\{x, y\}$ and relation $[x, y] = y$. Let V be a representation induced from a one-dimensional representation of the subalgebra spanned by x. Show that V is reducible. Does it have irreducible subrepresentations?

4. Prove that L is simple if $\dim(L) = 3$ and $L = [L, L]$.

5. Let L be a solvable Lie algebra. Show that the adjoint representation of L is completely reducible if and only if L is abelian.