Introduction to knot theory, Spring 2013

Homework 6, due Thursday, March 7

1. (a) Let $R = \mathrm{Mat}(n, F)$ be the algebra of n by n matrices with coefficients in a field F. Show that any trace-like map $\epsilon : R \longrightarrow F$ (an F-linear map with $\epsilon(ab) = \epsilon(ba)$ for all $a, b \in R$) is a multiple of the usual trace map on matrices.

(b) Classify all trace-like maps from TL_2 and TL_3 into the ground field $\mathbb{Q}(A)$ (use the idea about closing an element of TL_n into a diagram on a punctured plane and keeping track of circles that go around the puncture).

(b) The element u_2u_1 generates a left ideal $TL_3u_2u_1$. Determine the dimension of this ideal as a vector space over the ground field $\mathbb{Q}(A)$. This ideal is a left module over TL_3. Prove that this module is isomorphic to the module $V_{3,1}$ over TL_3 (Recall that in class we defined modules $V_{n,k}$ over TL_n for $k = n, n-2, \ldots$). Show that this module is simple.

2. In class we constructed a homomorphism $\psi : B_n \longrightarrow TL_n^*$. Show that the image $\psi(\theta_n)$ of the central braid $\theta_n = \Delta_n^2$ is central in TL_n. Compute $\psi(\theta_3)$ in the monomial basis of TL_3 (Hint: first compute $\psi(\Delta_3)$).

3. Draw a non-alternating diagram D with at least 5 crossings of some knot (preferably not the unknot). Determine diagrams s_+D and s_-D. What are the highest and lowest powers of A that these diagrams contribute to $< D >$? Do these powers survive in $< D >$ or get cancelled out?