Introduction to knot theory, Spring 2013

Read section 4 of Knotes.

Homework 5, due Thursday, February 28

1. Exercise 4.1.6 in Knotes (page 23).

2. Compute the Jones polynomial of the Hopf link via the Kauffman bracket definition.

3. Using the recursive relation, compute the Jones polynomial of the left-handed trefoil (closure of braid σ_1^{-3}). Via the symmetry, determine the Jones polynomial of the right-handed trefoil and show that trefoil is not isotopic to its mirror image.

4. Compute the Jones polynomial $J(4_1)$ of the figure-eight knot via the skein relation. Is the Jones polynomial self-conjugate (invariant under the substitution $q \to q^{-1}$?

5. Prove that $J(K_1 \# K_2) = J(K_1)J(K_2)$ and

$$J(K_1 \sqcup K_2) = J(K_1)J(K_2)(-q - q^{-1}).$$

Here $K_1 \sqcup K_2$ is the disjoint union of links K_1 and K_2. Show that the knots $3_1 \# 3_1$ and $3_1 \# 3_1^!$ are distinct.

Read "Catalan numbers", by Tom Davis.

6. Draw all crossingless matchings with 8 endpoints. Write down all elements in the monomial basis of TL_4. There must be a natural bijection between these elements and crossingless matchings.

Extra credit: Problem in section 7.2 of "Catalan numbers" - show that there are exactly c_n ways to tile the staircase with n steps by n rectangles.