Introduction to knot theory, Spring 2012

Homework 3, due Thursday, February 14

1. Describe a braid whose closure is (a) the $(2, n)$-torus knot, (b) (n, m)-torus knot.

2. In class we described configuration spaces C'_n and C_n. Set $n = 2$ and describe explicitly C'_2 and C_2. What are the fundamental groups of these spaces? Look for simplest spaces homotopy equivalent to C'_2 and C_2. Can you show that the universal cover of C_2 is contractible?

3. We constructed an action of the braid group B_n on the free group F_n with generators $x_1, \ldots, x_n$ via

$$\begin{aligned} \sigma_i(x_j) &= x_j, \quad \text{if } j \neq i, i+1, \\ \sigma_i(x_i) &= x_{i+1}, \\ \sigma_i(x_{i+1}) &= x_{i+1} x_i x_{i+1}^{-1}. \end{aligned}$$

(a) Write down the action of σ_i^{-1} on generators $x_1, \ldots, x_n$.
(b) Check directly that these formulas give an action of the braid group by confirming that the defining relations in the braid group hold for this action.

4. Let B'_n be the subgroup of B_n of braids in which the leftmost strand is "pure" (begins and ends in position 1). Find the center of B'_n for $n \geq 4$. Is B'_n a normal subgroup of B_n?

5. Draw the 3-braid $\sigma_1 \sigma_2^{-2} \sigma_1$. Check that it is a pure braid and compute or draw its normal (combed) form. Do the same for the 3-braid $\sigma_1^2 \sigma_2^{-2} \sigma_1^{-2}$.