Algebraic topology, Fall 2014

Homework 4, due Monday, October 13

1. Show that $\mathbb{S}^2$ and $\mathbb{S}^3 \times \mathbb{CP}^\infty$ have isomorphic homotopy groups but are not homotopy equivalent.

2. Prove that, if spaces X and Y are weakly homotopy equivalent, there exist a CW-complex Z and maps $Z \longrightarrow X$, $Z \longrightarrow Y$ that are weak homotopy equivalences.

3. Show that the Whitehead product $[\alpha, \beta] = \alpha\beta\alpha^{-1}\beta^{-1}$ if $\alpha, \beta \in \pi_1(X)$.

4. Prove that for $\alpha \in \pi_n(X), \beta \in \pi_m(X)$

$$[\alpha, \beta] = (-1)^{nm}[\beta, \alpha]$$

for $\alpha \in \pi_n(X), \beta \in \pi_m(X)$.

5. Exercise 10.6 on page 81 of Botvinnik's "Lecture notes on Algebraic Topology" (there's a link to it in "Additional resources" section of our webpage).

6. (Exercise 10.10 from Botvinnik's notes) Prove that the suspension $\Sigma(S^n \times S^k)$ is homotopy equivalent to the wedge

$$S^{n+1} \vee S^{k+1} \vee S^{n+k+1}.$$