Representations of finite groups, spring 2016
Homework 7, due Wednesday March 9 before class.

Reading: Chapter 6 of Steinberg (pages 71-82). Cyclotomic polynomials are also discussed in Dummit and Foote, Chapter 13.6.

1. (15 points, this is essentially exercise 6.2 in Steinberg).

(a) Describe explicitly the unique (up to isomorphism) non-abelian group G of order 39 as a subgroup of 2-by-2 invertible matrices with coefficients in the field $\mathbb{F}_{13}$. (We discussed in class how to do this for groups of order pq, where p and p are primes, with $p < q$ and $q \equiv 1 (\mathrm{mod}\ p)$.)

(b) Putting together all the results we've discovered about degrees of irreducible representations of finite groups, determine the degrees of irreducible representations of G. How many irreducible representations does G have in each degree (up to isomorphism)?

(c) Determine the number of conjugacy classes of G.

2. (10 points) Show that if $\phi : G \longrightarrow GL(V)$ is a d-dimensional representation, then $g \in \ker\phi$ if and only if $\chi_V(g) = d$.

3.(10 points) We proved that dimension d of an irreducible complex representation of a finite group G divides the order of G. Does this hold for irreducible real representations as well, that is, if V is an irreducible representation of G over $\mathbb{R}$ then $\dim(V)$ divides $|G|$? Either prove or give a counterexample.

4.(20 points) Consider the dihedral group D_5 of order 10.

(a) Recall and write down generators and relations of D_5, when viewed as the group of symmetries of a regular pentagon. What is the number of conjugacy classes?

(b) What is the commutator subgroup $[D_5, D_5]$? Determine all one-dimensional representations of D_5. Explain why the fundamental representation of D_5 is irreducible, even over $\mathbb{C}$.

(c) Determine the number and dimensions of irreducible representations of D_5.

(d) Write down the character table of D_5.

5. (5 points) Prove that if a complex number α is algebraic, that is, $\alpha \in \overline{\mathbb{Q}}$, then $n\alpha$ is an algebraic integer for some integer $n > 1$.