Representations of finite groups. Practice Midterm.

1. Mark those squares below that are followed by correct statements. Representations are assumed to be over complex numbers, unless specified otherwise.

☐ Any representation of a finite group is completely reducible.

☐ If a finite group is simple, it has only one irreducible representation.

☐ Any two representations of the trivial group are isomorphic.

☐ The cyclic group $\mathbb{Z}_n$ has exactly n pairwise non-isomorphic irreducible representations.

☐ Any representation of $\mathbb{Z}$ is completely reducible.

☐ The group S_5 has 5 conjugacy classes.

☐ Any irreducible representation of a finite group G is isomorphic to a direct summand of the regular representation.

☐ $\langle \chi_{reg}, \chi_V \rangle = |G|$ for any representation V of a finite group G.

☐ If $\langle \chi_V, \chi_V \rangle = 1$ then V is irreducible.

☐ The symmetric group S_3 has an irreducible representation of dimension 3.

2. Write down the character table of the Klein four group $\mathbb{Z}_2 \times \mathbb{Z}_2$.

3. Give the definition of an irreducible representation of G.

4. Suppose that V is an irreducible non-trivial representation of the alternating group A_5. Show that V is faithful.

5. Let χ be the character of the 5-dimensional permutation representation of S_5. Find $\chi(x)$ for $x = (23)$ and $x = (54321)$.

6. Show that if every irreducible representation of a finite group G is one-dimensional then G is abelian.