

Representations of finite groups

Homework #4, due Wednesday, October 7.

1. Check that quaternionic conjugation $\overline{a + bi + cj + dk} = a - bi - cj - dk$ is an antiautomorphism of the ring \mathbb{H} of quaternions, in particular, $\overline{(q_1 q_2)} = \overline{q_2} \cdot \overline{q_1}$. Conclude that $\mathbb{H}^{op} \cong \mathbb{H}$.
2. Show that a division ring has only two idempotents: 0 and 1.
3. Determine which elements in the following rings are idempotents:

$$\mathbb{Z}/10, \quad \mathbb{Z}/12, \quad \mathbb{C}[x]/(x^2 - 1).$$

Observe that all these rings are commutative, so that your idempotents are central. Determine corresponding decompositions of the rings as direct products.

4. Show that the ring $\mathbb{C}[x]/(x^n)$ has only the trivial idempotents: 0 and 1.

5*(optional). Show that $\{0\}$ is the only 2-sided ideal of the matrix algebra $\text{Mat}_n(F)$ with coefficients in a field F .