**Homework 2, due Wednesday Sept 23.**

NAME:

Mark the squares that are followed by correct statements.

☐ If $M$ is a submodule of $N$ and $N$ a submodule of $K$, then $M$ is a submodule of $K$.

☐ Any finitely-generated module over a ring $R$ is a quotient of a free module $R^n$.

☐ The $\mathbb{Q}[x]$-module $\mathbb{Q}[x]/(x^2 - x)$ is simple.

☐ The $\mathbb{C}[x]$-module $\mathbb{C}[x]/(x + \sqrt{2})$ is simple.

☐ The $\mathbb{Z}$-module $\mathbb{Z}/33\mathbb{Z}$ is simple.

☐ Direct sum of two simple modules is simple.

☐ Any division ring is commutative.

☐ A module over a field is cyclic if and only if it is simple.

☐ If a module has no proper submodules, it is simple.

☐ Any left ideal of a ring $R$ is a left $R$-module.

☐ Direct sum of two cyclic modules is cyclic.

☐ A quotient module of a cyclic module is cyclic.

2. Give an example of a $\mathbb{Z}$-module which has exactly three proper submodules.