Representations of finite groups

Homework #12, due Wednesday, December 9.

1. Determine the number of conjugacy classes in each finite subgroup of $SU(2)$ and in each finite subgroup of $SO(3)$ (hint: don't use brute force).

2. Dihedral group D_{2n} is isomorphic to the quotient of binary dihedral group D^*_{2n} by the central subgroup $\{I, -I\}$. Use the properties of the affine graph of D^*_{2n} to compute the number of irreducible representations of D_{2n} of dimension 2.

3. (a) If G is a finite subgroup of $SU(2)$ and $V \cong \mathbb{C}^2$ the defining 2-dimensional representation, show that $\Lambda^2(V)$ is the trivial representation.

(b) Using decomposition $V \otimes V = S^2(V) \oplus \Lambda^2(V)$ and properties of the affine graph of G, find for which groups G the representation $S^2(V)$ is irreducible.

4. For each finite $G \subset SU(2)$ determine the abelian group $G/[G, G]$.