Name: ____________________

Introduction to algebraic topology. Midterm exam. March 14, 2013

You can solve problems in any order. Textbooks and notebooks are not allowed on the exam.

1. (15 points) Mark those squares that are followed by correct statements.

☐ If Y is a retract of X and Z a retract of Y then Z is a retract of X.

☐ Any two maps from a circle to the punctured 3-space $\mathbb{R}^3 \setminus \{p\}$ are homotopic (here p is a point in $\mathbb{R}^3$).

☐ If two graphs are homotopy equivalent, they are homeomorphic.

☐ Any finite CW-complex is homotopy equivalent to a simplicial complex.

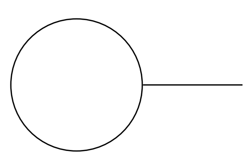
☐ Identity map $\mathrm{Id} : X \longrightarrow X$ is a covering map for any topological space X.

☐ If $\pi_1(X) = C_3$ then X is contractible (here C_3 is the cyclic group of order 3).

☐ Any fiber bundle over the interval $[0, 1]$ is trivial.

☐ Any fiber bundle over the circle S^1 is trivial.

2. (20 points) Let X be a circle with an interval attached.



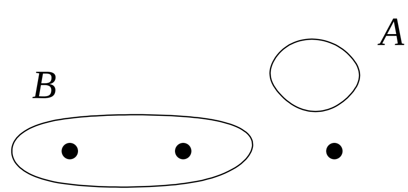
What is the fundamental group of X? Describe the universal covering space $\widehat{X}$. Is $\widehat{X}$ contractible? Can $\widehat{X}$ be turned into a CW-complex? into a simplicial complex? Is the covering $\widehat{X} \longrightarrow X$ regular?

3. (30 points) Determine fundamental groups of the following spaces.

I. Direct product $\mathbb{RP}^2 \times MB$ of a projective plane $\mathbb{RP}^2$ and a Möbius band MB.

II. Two 2-spheres with equatorial circles identified.

III. A triply-punctured plane $\mathbb{R}^2 \setminus \{p_1, p_2, p_3\}$ to which a 2-cell is attached along (a) loop A, (b) loop B. In the picture below the punctures are shown as dots.



4. (25 points) (a) State the definition of a fiber bundle (E, F, B, p).

(b) Briefly describe the exact homotopy sequence of a fiber bundle and where the maps in the sequence come from.

(c) Consider the Hopf bundle with total space S^3, fiber S^1 and base S^2. How does the sequence simplify in this case and what does it tell us about the relation between the homotopy groups of S^2 and S^3?

Extra credit. Give an example of a space X with $\pi_1(X) = \mathbb{Z}$ and $\pi_2(X) = \mathbb{Z}$.