Introduction to algebraic topology, Spring 2013

Homework 9, due Tuesday, April 9

Hatcher, Section 2.1 (page 131) exercises 4, 5 (in exercise 5 Hatcher is referring to page 102; also look at his examples 2.2-2.5 on page 106), 11, 12.

1. (a) Show that the sum $f+g$ and difference $f-g$ of null-homotopic morphisms $f, g : A \longrightarrow B$ of complexes are null-homotopic.

(b) Suppose that $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{k} D$ are maps of complexes and g is null-homotopic, $g \sim 0$. Show that gf and kg are null-homotopic as well.

2. Show that there exists a short exact sequence

$$0 \longrightarrow \mathbb{Z}_4 \longrightarrow \mathbb{Z}_8 \oplus \mathbb{Z}_2 \longrightarrow \mathbb{Z}_4 \longrightarrow 0$$

by constructing suitable differentials between these abelian groups.

3. Compute simplicial homology groups of (a) the 1-skeleton of a 3-simplex, (b) the 2-skeleton of a 3-simplex.