Introduction to algebraic topology, Spring 2013

Homework 8, due Tuesday, April 2

1. (10 points) Suppose that complex C is the direct sum of complexes A and B. Prove that $H_n(C) \cong H_n(A) \oplus H_n(B)$ for all n.

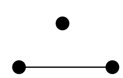
2. (30 points) Consider the following complexes

$$\begin{array}{ll}
A & 0 \longrightarrow \mathbb{Z} \xrightarrow{0} \mathbb{Z} \longrightarrow 0 \\
B & 0 \longrightarrow \mathbb{Z} \xrightarrow{2} \mathbb{Z} \longrightarrow 0 \\
C & 0 \longrightarrow 0 \longrightarrow \mathbb{Z}/2 \longrightarrow 0 \\
D & 0 \longrightarrow \mathbb{Z}/4 \xrightarrow{-1} \mathbb{Z}/4 \longrightarrow 0
\end{array}$$

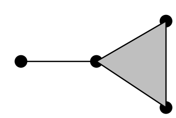
Assume that the rightmost term in each complex is in degree 0. In which degrees are these complexes nontrivial? Compute homology groups of these complexes. Determine all possible homomorphisms of complexes $\mathrm{Hom}(A, B)$, $\mathrm{Hom}(B, C)$, $\mathrm{Hom}(C, B)$, $\mathrm{Hom}(C, D)$, $\mathrm{Hom}(D, C)$. Which of these homomorphisms induce nontrivial maps on homology groups?

3. (30 points) For each of simplicial complexes below, write down its chain complex of simplicial chain groups. In which degrees is the complex nontrivial? Compute homology groups of these complexes.

(a)



(c)



(b)